Exercise 14

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Homework Problem 14.1. (Numerical Optimization of the Floor Heating Problem)

We will treat the following specialized version of the floor heating problem

$$\begin{aligned} \text{Minimize} \quad & \frac{1}{2}\|y - y_d\|^2_{L^2([0,1]\times[0,1])} + \frac{\gamma}{2}\|u\|^2_{L^2([0,1]\times[0,1])} \\ \text{s. t.} \quad & \begin{cases} -\operatorname{div}(\nabla y) = u & \text{in } [0,1]\times[0,1] \\ \dfrac{\partial}{\partial n} y = -y & \text{on } \partial([0,1]\times[0,1]) \end{cases} \end{aligned} \tag{P}$$

numerically.

(a) Identify and state, which parameters of the floor heating problem (7.3) have been specified to which value to obtain (P), and address which quantities remain to be fixed, before a concrete solution can be computed.

(b) Install Fenics (not FenicsX). Download the file solve_KKT_sparse.py. The file contains a python solution for solving an FEM approximation of (P) for multiple regularization parameters γ in two problem settings where the desired states y_d are computed by solving the PDE for provided u_d. The results are displayed in subplots of figures. When you run the file, you should see the desired states plotted but all other subplots empty because the code segment that solves the optimization problems has been removed. Read and fix the code (lines 77 and 80) so the optimization problems are solved and give a quick analysis of what you can observe from the solutions for various γ's.

You are not expected to turn in your solutions.