R. Herzog, G. Müller
Universität Heidelberg

Exercise 13

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Homework Problem 13.1.

Let U, H be Hilbert spaces and $S \in \mathcal{L}(U, H)$ as well as $\gamma \geqslant 0$ be a real number. Show Lemma 11.1 of the lecture notes, i. e., that the operator $S^\circ S + \gamma\,\mathrm{id}$ is self-adjoint and positive semi-definite and positive definite, if $\gamma > 0$.

Homework Problem 13.2.

Consider the quadratic objective

$$f(u) = \underbrace{\frac{1}{2}(A\,u, u)_U}_{\text{quadratic}} - \underbrace{(b, u)_U}_{\text{linear}} + \underbrace{c}_{\text{constant}} \tag{11.1}$$

from the lecture notes. Show Lemma 11.2 of the lecture notes, i. e., the identities:

$$f(u + \alpha\,d) = f(u) + \alpha\,\big(\underbrace{A\,u - b}_{=\nabla f(u)}, d\big)_U + \frac{\alpha^2}{2}(A\,d, d)_U \tag{11.4a}$$

$$\frac{\mathrm{d}}{\mathrm{d}\alpha} f(u + \alpha\,d) = \big(\nabla f(u + \alpha\,d), d\big)_U. \tag{11.4b}$$

You are not expected to turn in your solutions.