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Exercise 11

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Homework Problem 11.1 (QP Reformulation) 3 Points

Show that solving (9.7) and using the associated Lagrange multiplier (as described in (9.8)) leads to the same next iterate $(x^{(k+1)}, \lambda^{(k+1)})^\mathsf{T}$ as solving the "original" QP (9.5).

Homework Problem 11.2 (Complementarity is equivalent to variational inequality) 3 Points

Prove Lemma 9.4 of the lecture notes, i. e. the equivalence of the KKT complementarity condition

$$\mu \geq 0, \quad g(x) \leq 0, \quad \mu^\mathsf{T} g(x) = 0 \tag{9.11b}$$

and the variational inequality

$$\mu \in K \quad \text{and} \quad g(x)^\mathsf{T}(v - \mu) \leq 0 \quad \text{for all } v \in K \tag{9.12}$$

with the closed convex cone $K := \mathbb{R}^{n_{\text{ineq}}}_{\geq 0}$ (the non-negative orthant).

Homework Problem 11.3 (On the normal cone) 3 Points

Prove Lemma 9.6 of the lecture notes, i. e., the following statements for a set $M \subseteq \mathbb{R}^n$ and $x \in M$:

(*i*) The normal cone is a closed convex cone.

(*ii*) $\mathcal{N}_M(x) = (M - \{x\})^\circ$ holds.

Additionally, prove that

(*iii*) $\mathcal{N}_M(x) \subseteq \mathcal{T}_M(x)^\circ$ but generally $\mathcal{N}_M(x) \neq \mathcal{T}_M(x)^\circ$.

Homework Problem 11.4 (Examples for generalized Newton) 5 Points

For the nonlinear functions $F\colon \mathbb{R} \to \mathbb{R}$ and the set valued functions $N\colon \mathbb{R} \to \mathcal{P}(\mathbb{R})$ below, find all solutions z^* of the generalized equation

$$0 \in F(z) + N(z)$$

and determine, at which solutions the problem is strongly regular.

(*i*) $F(z) \coloneqq z^2 - 1$ and $N(z) \coloneqq \{0\}$

(*ii*) $F(z) \coloneqq z^2 - 1$ and $N(z) \coloneqq \mathbb{R}_{\geq}$

(*iii*) $F(z) \coloneqq (z-1)^2$ and $N(z) \coloneqq \mathcal{N}_{\mathbb{R}_{\geq}}(z)$

Please submit your solutions as a single pdf and an archive of programs via moodle.