Exercise 6

Date issued: 21st May 2024
Date due: 28th May 2024

Homework Problem 6.1 (Example for convergence of the local Newton's method) 6 Points

Let $p > 2$ and $f : \mathbb{R} \to \mathbb{R}, f(x) \coloneqq |x|^p$ be given. Consider the local Newton's method (Algorithm 4.23) for minimization of f, i.e. $F(x) = \nabla f(x)$, with some initial guess $x^{(0)} > 0$.

(i) Show that the method converges to the global minimizer $x^* = 0$ of f.

(ii) Which rate of convergence do you observe?

(iii) Why is this result not in contradiction with Theorem 4.27?

Homework Problem 6.2 (On the Restriction $\sigma \in (0, \frac{1}{2})$ in Globalized Newton) 7 Points

In the globalized Newton's method for optimization (Algorithm 4.30 of the lecture notes), the Armijo-parameter, which is typically chosen as $\sigma \in (0, 1)$, is restricted to the interval $(0, \frac{1}{2})$ so that the full Newton step size $\alpha^{(k)} = 1$ can in fact be accepted by the Armijo condition for $k \geq k_0$ and some $k_0 > 0$, in order to facilitate quadratic convergence in the final stages of the algorithm. We will investigate why that is:

(i) Show that the step length $\alpha^{(k)} = 1$ satisfies the Armijo condition for the Newton direction $d^{(k)} \neq 0$ for the quadratic function

$$f(x) = \frac{1}{2} x^\mathsf{T} A x + b^\mathsf{T} x + c$$

with s. p. d. $A \in \mathbb{R}^{n \times n}$, $b \in \mathbb{R}^n$, und $c \in \mathbb{R}$ if and only if $\sigma \leq \frac{1}{2}$.

(ii) Explain why we need to restrict ourselves to $\sigma < \frac{1}{2}$ for general nonquadratic problems.

Homework Problem 6.3 (Characterization of fast local convergence) 6 Points

The proof of Lemma 4.36 is given in the lecture notes. Your task is to carefully read and understand the proof. Then write it down in your own words.

Homework Problem 6.4 (Globalized Newton's Method in Optimization) 8 Points

Implement the globalized Newton's method for optimization (Algorithm 4.30 of the lecture notes), run it for the Rosenbrock's and/or Himmelblau's functions and compare its performance to that of your gradient descent implementation.

Please submit your solutions as a single pdf and an archive of programs via moodle.