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Exercise 4

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Homework Problem 4.1 (Angle condition implies admissibility) 2 Points

Prove Lemma 4.4 from the lecture notes, i.e. if the angle condition (4.8) holds with some $\eta \in (0,1)$, then the sequence $d^{(k)}$ of search directions is admissible.

Homework Problem 4.2 (Efficient Step Sizes in Quadratic Steepest Descent) 4 Points

Show that both constant step sizes (as in § 3.3) as well as the Cauchy step size are efficient for the steepest descent method (cf. Algorithm 3.6) for solving quadratic optimization problems of the type

$$\text{minimize} \quad f(x) := \frac{1}{2} x^\mathsf{T} A x - b^\mathsf{T} x + c \quad \text{where } x \in \mathbb{R}^n$$

with s. p. d. $A \in \mathbb{R}^{n \times n}$, $b \in \mathbb{R}^n$, $c \in \mathbb{R}$.

Homework Problem 4.3 (Efficiency implies admissibility) 2 Points

Prove Lemma 4.8, i.e. if the sequence of step sizes $\alpha^{(k)}$ is efficient, then it is also admissible.

Homework Problem 4.4 (Armijo steplength is not admissible in general) 6 Points

Show that the Armijo step sizes are not admissible in general.
Consider the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) := \frac{x^2}{8}$ with $M = 1$, admissible search directions $d^{(k)} = -2^{-k} f'(x^{(k)})$ and initial trial step size $\alpha^{(k,0)} = 1$ in the Armijo backtracking line search Algorithm 4.11.

Hint: Show that for any starting point $x^{(0)} > 0$ the generated sequence $(x^{(k)})$ by Algorithm 4.2 is monotonically decreasing, but converges to $\bar{x} \geq \frac{x^{(0)}}{2}$. Then explain, why $\bar{x}$ can not be a stationary point and how this shows that the Armijo step sizes are not admissible.

Please submit your solutions as a single pdf and an archive of programs via moodle.