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Exercise 4

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Homework Problem 4.1 (Efficient Step Sizes in Quadratic Steepest Descent) 4 Points

Show that both constant step sizes (as in § 4.3) as well as the Cauchy step size are efficient for the steepest descent method (cf. Algorithm 4.6) for solving quadratic optimization problems of the type

$$\text{minimize} \quad f(x) := \frac{1}{2} x^\mathsf{T} A x - b^\mathsf{T} x + c \quad \text{where } x \in \mathbb{R}^n$$

with s. p. d. $A \in \mathbb{R}^{n \times n}$, $b \in \mathbb{R}^n$, $c \in \mathbb{R}$.

Homework Problem 4.2 (Efficiency of Wolfe-Powell Step Sizes for $C^{1,1}$ Functions) 5 Points

Let $f \in C^1$ and let $x^{(0)} \in \mathbb{R}^n$ be an initial iterate of the generic descent scheme (Algorithm 5.2). Further assume that f' is Lipschitz continuous on the sublevel set $\mathcal{M}_f(x^{(0)}) := \{x \in \mathbb{R}^n \mid f(x) \le f(x^{(0)})\}$.

Show that step sizes $\alpha^{(k)}$ that satisfy the Wolfe-Powell-conditions at $x^{(k)}$ for the descent direction $d^{(k)}$ for all k are efficient and that there is a $c > 0$ such that

$$f(x^{(k)} + \alpha^{(k)} d^{(k)}) - f(x^{(k)}) \le -c \left(\cos \measuredangle(-\nabla_M f(x^{(k)}), d^{(k)}) \|f'(x^{(k)})^\mathsf{T}\|_{M^{-1}} \right)^2$$

for all $k \ge 0$.

Homework Problem 4.3 (Scaling Invariance of Armijo- and Curvature Conditions) 5 Points

Show the statement of remark 5.21, i. e. that when a step length α satisfies any of the Armijo- or curvature conditions (5.12), (5.17) and (5.18) for $g(x) := \gamma f(Ax + b) + \delta$ at $x \in \mathbb{R}^n$ with search direction $d \in \mathbb{R}^n$, where $A \in \mathbb{R}^{n \times n}$ is non-singular, $b \in \mathbb{R}^n$, $\gamma > 0$ and $\delta \in \mathbb{R}$, then it satisfies the respective conditions for f at $Ax + b$ with the search direction Ad.

Homework Problem 4.4 (Implementation of Nonlinear Steepest Descent and Armijo Backtracking)
8 Points

Implement the M-steepest descent method as outlined in Algorithm 5.22 with the original and the modified (interpolating) Armijo backtracking as outlined in Algorithms 5.11 and 5.15.

Visualize and examine the effect of the parameters of the step size strategy on the behavior of the algorithm when applied to quadratic, strongly convex functions, Rosenbrock's and/or Himmelblau's functions.

Please submit your solutions as a single pdf and an archive of programs via moodle.