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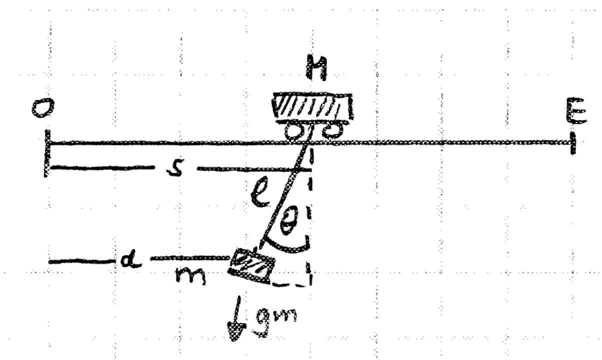
Exercise 4

Problem 4.1. (Optimal Control of a hanging pendulum)

In this exercise, we want to investigate the linearized version of the control-constrained one dimensional problem of time/cost-optimally controlling a load on a hanging pendulum connected to a carriage system by a rod from an initial rest state to a final rest state. We consider being able to control the acceleration of the carriage (through forces). You can think of an application of a storage unit crane moving a load around.



The model is described by the following information.

(i) M is the mass of the carriage

(ii) m is the mass of the load hanging from the carriage

(iii) s is the x-displacement of the carriage

(iv) d is the x-displacement of the load

(v) Θ is the angle of the rod at the carriage relative to vertical

(vi) E is the final x-position that both the carriage and the load are supposed to end up in rest at.

(vii) The state variables of this problem are $x = (s, \dot{s}, z, \dot{z})$ where z is the x-position of the load relative to the carriage.

The corresponding optimal control problem reads as

$$\begin{aligned} \text{minimize} \int_0^T 1 + \frac{\gamma}{2} u(t)^2 \,\mathrm{d}t \quad & \text{with respect to } (u, T) \in L^2(0, T) \times \mathbb{R} \\ \text{s.t. } \dot{x}(t) &= Ax(t) + Bu(t) \\ x(0) &= 0 \\ x(T) &= (E, 0, 0, 0) \\ u(t) &\in (-1, 1) \\ T &> 0 \end{aligned}$$

for

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{g}{l} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1/M \\ 0 \\ 1/M \end{bmatrix}$$

and the regularization parameter $\gamma > 0$

Discretize the problem using piecewise constant controls, solve the problem numerically and investigate the behavior of the control and the influence of the problem parameters.