Mixed−Integer Optimization
Optimization under Uncertainty
Optimization in Applications
Shape and Topology Optimization
Optimization and Machine Learning
2023
Heidelberg
EUCCO
6th European Conference on Computational Optimization
Optimal and Feedback Control of PDEs Optimization on Manifolds
Non−smooth Optimization Inverse Problems Model−Order Reduction for Optimization

6TH EUROPEAN CONFERENCE ON CONTINUOUS OPTIMIZATION 2023

BOOK OF ABSTRACTS

EUCCO Organizers*

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- Heidelberg University
- Interdisciplinary Center for Scientific Computing (IWR)
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enabling us to offer travel support to young scientists and to keep the registration fee for all participants to a minimum. Their contributions are greatly appreciated.

Contents

GENERAL INFORMATION ON PRESENTATIONS

The 2023 edition of the European Conference on Continuous Optimization features 2 plenary lectures of 50 minutes plus questions and answers each. The plenary lecturers are (ordering by last name):

- Frank E. Curtis (Lehigh University)
  Stochastic Algorithms for Continuous Optimization with Nonlinear Constraints
- ~~Carola-Bibiane Schönlieb (University of Cambridge)~~
  ~~What can we learn from deep learning? From learned optimisation schemes to stochastic optimisation for tomographic image reconstruction~~ **(Talk cancelled)**

Additionally, there will be a total of 129 presentations of 30 minutes including questions and answers each. These presentations are organized in the following 10 focus sessions:

1. ● Inverse Problems (IP) — 11 speakers
   organized by Anne Wald and Tram Nguyen

2. ● Mixed-Integer Optimization (MIO) — 8 speakers
   organized by Paul Manns

3. ● Model-Order Reduction for Optimization (MOR) — 9 speakers
   organized by Carmen Gräßle and Sebastian Peitz

4. ● Non-smooth Optimization (NSO) — 14 speakers
   organized by Anne-Therese Rauls and Kathrin Welker

5. ● Optimal and Feedback Control of PDEs (PDE) — 16 speakers
   organized by Daniel Walter and Hannes Meinlschmidt

6. ● Optimization and Machine Learning (ML) — 17 speakers
   organized by Olga Mula and Aymeric Dieuleveut

7. ● Optimization in Applications (APP) — 18 speakers
   organized by Claudia Totzeck and Dante Kalise

8. ● Optimization on Manifolds (MAN) — 10 speakers
   organized by Thomas Bendokat and Estelle Massart

9. ● Optimization under Uncertainty (OU) — 12 speakers
   organized by Caroline Geiersbach and Aswin Kannan

10. ● Shape and Topology Optimization (STO) — 14 speakers
    organized by Peter Gangl and Estefanía Loayza

1 Inverse Problems (IP)

OPTIMAL DESIGN FOR AEROACOUSTICS WITH CORRELATION DATA

Christian Aarset
Georg-August Universität Göttingen

A key problem in aeroacoustics is the inverse problem of estimating an unknown random source from correlation data sampled from surrounding sensors. We study optimal design for such problems, that is, identifying the sensor placement minimising covariance of the solution to the inverse random source problem, while remaining sparse. Explicitly, we show existence and further properties of

$$\operatorname{argmin}_{w,\,0\leq w\leq 1} \operatorname{tr}(C(w)) + \alpha\|w\|_1,$$

where $w \in L^\infty(\Omega)$ represents potential sensor placements, $C(w)$ represents the posterior covariance of the inverse problem as a function of sensor placement, and the term $\alpha\|w\|_1$ enforces sparsity of the sensor placement.

PHYSICS-INFORMED NEURAL NETWORKS FOR MATERIAL MODEL CALIBRATION IN STRUCTURAL HEALTH MONITORING

David Anton
Technische Universität Braunschweig

The identification of material parameters occurring in material models is essential for structural health monitoring (SHM). The material parameters indicate possible damage and material degradation, as they directly reflect the resistance of the structure to external impacts. We further developed physics-informed neural networks (PINNs) for the calibration of the linear- elastic material model from full-field displacement data and global force data in a realistic regime. For a realistic data regime, the optimization problem had to be conditioned. However, this approach is computationally expensive and prone to realistic noise levels in the measurement data. In our ongoing work, we are therefore focusing on learning parameterized solutions of parametric partial differential equations. We further investigate the ability of parametric PINNs to act as a surrogate for the identification of material parameters from full-field displacement data.

A trust-region algorithm for parameter identification in multigroup pandemic models

Markus Friedemann
BTU Cottbus Senftenberg

Parameter identification for a nonlinear ODE describing a pandemic model is studied. In the model, the population is divided into several spatially separated subpopulations. Between those subpopulations, there is some (possibly infectious) contact due to traveling. The main goal is the identification of time-dependent contact parameters in the interaction terms between subgroups. The parameters can be considered in $L^2(0,T)$ and $H^1(0,T)$ to provide the possibility of regularizing the solution in the case of noisy data. Existence of solutions is proven and optimality conditions are discussed. A penalty approach is chosen to handle constraints. A solution algorithm based on a trust-region approach with Steihaug CG as an inner solver is introduced. Global and local convergence properties are investigated and fast local convergence is proven. Numerical results are presented. The quality of the identification for the parameters is analyzed.

TIKHONOV REGULARIZED ITERATIVE METHODS FOR NONLINEAR PROBLEMS

Pankaj Gautam
Norwegian University of Science and Technology (NTNU)

In this work, we propose a new common fixed point algorithm based on normal S-iteration method using Tikhonov regularization to find common fixed points of nonexpansive operators and prove strong convergence of the generated sequence to the set of common fixed points. Based on the proposed fixed point algorithm, we propose a forward-backward-type algorithm and a Douglas-Rachford algorithm in connection with Tikhonov regularization to find the solution to monotone inclusion problems. Further, we consider the complexly structured monotone inclusion problems which are very popular these days. We also propose a strongly convergent forward-backward-type primal-dual algorithm and a Douglas-Rachford-type primal-dual algorithm to solve the monotone inclusion problems. Finally, we conduct a numerical experiment to solve image deblurring problems.

STEMPO – DYNAMIC X-RAY TOMOGRAPHY PHANTOM

Tommi Heikkilä
University of Helsinki

Tomography is an imaging modality with a range of applications. Novel inverse problems research can still push the boundaries of what computed tomography is capable. For example in dynamic tomography the object of interest changes during the tomographic measurement process causing reconstruction errors and motion artifacts if traditional methods are used. In recent years active research has lead to many promising methods to overcome this. However openly available dynamic data for assessing and comparing these methods is still lacking.

The Spatio-TEmporal Motor-POwered (STEMPO) phantom is a computer controlled mechanical device designed for collecting dynamic X-ray tomography data using various measurement setups. The device itself and the currently available data are briefly described and to illustrate its capabilities, some examples are given using reconstruction methods such as sparse + low-rank matrix decomposition and spatio-temporal wavelet regularization.

TOWARDS OPTIMAL SENSOR PLACEMENT FOR SPARSE INVERSE PROBLEMS

Phuoc Truong Huynh
Alpen-Adria-Universität Klagenfurt

In this talk, we study parameter identification problems from a finite number of measurements under a sparsity assumption. Since the data are contaminated by Gaussian noise, a statistical framework for their recovery is considered. It relies on two main ingredients: first, a convex but nonsmooth Tikhonov point estimator over the space of Radon measures, and second, a suitable mean-squared error based on its Hellinger-Kantorovich (H-K) distance to the ground truth.

Assuming standard non-degenerate source conditions and applying careful linearization arguments, we derive a sharp upper bound for the H-K distance between the aforementioned ground truth and an estimator. On the one hand, this allows us to derive asymptotic convergence results for the mean-squared error, which is later used as a crucial tool for the sensor placement problem. Finally, we present some numerical results to illustrate our theory.

ACOUSTIC NONLINEARITY PARAMETER TOMOGRAPHY AS AN INVERSE COEFFICIENT PROBLEM FOR A NONLINEAR WAVE EQUATION

Barbara Kaltenbacher
Alpen-Adria-Universität Klagenfurt

We consider an undetermined coefficient inverse problem for a nonlinear partial differential equation occuring in high intensity ultrasound propagation as used in acoustic tomography. In particular, we investigate the recovery of the nonlinearity coefficient commonly labeled as B/A in the literature, which is part of a space dependent coefficient κ in the Westervelt equation governing nonlinear acoustics. Corresponding to the typical measurement setup, the overposed data consists of time trace measurements on some zero or one dimensional set Σ representing the receiving transducer array. In this talk, we will dwell on topics like modeling, uniqueness, numerical reconstruction schemes (in particular based on Newton type methods) as well as simultaneous reconstruction of κ and the sound speed. Finally we will also show some numerical reconstructions based on a formulation of this problem in frequency domain. This is joint work with Bill Rundell, Texas A&M Univ.

A TIME-DEPENDENT EXTENSION OF THE INVERSE PROBLEM IN 3D DIFFUSIVE OPTICAL TOMOGRAPHY

Mariella Kast

We propose a time-dependent extension of the inverse problem in 3D diffusive optical tomography (DOT): instead of reconstructing the target from a single light intensity measurement of the steady state, we explicitly model the dynamics of the injected tracer and consider a time-series of snapshots in the transient regime. This increase in observation data allows for better reconstruction quality, so that tumors can be detected at larger depth. In this talk, we discuss appropriate choices for the dynamic model and the algorithmic challenges of extending the optimisation process to a large number of snapshots. While gradients can be efficiently computed via the adjoint equations, both the single and multi-snapshot cases are ill-posed and require appropriate regularisation and constraints. We further show how Bayesian uncertainty quantification can be used to assess the reliability of the optimisation result and to estimate the limit of reconstruction depth for an experimental setup.

EXACT PARAMETER IDENTIFICATION IN PET PHARMACOKINETIC MODELING USING THE IRREVERSIBLE TWO TISSUE COMPARTMENT MODEL

Erion Morina
University of Graz

In this talk we consider the identifiability of metabolic parameters from multi-region measurement data in quantitative PET imaging. We discuss how, for the frequently used two-tissue compartment model and under reasonable assumptions, it is possible to uniquely identify metabolic tissue parameters from standard PET measurements, without the need of additional concentration measurements from blood samples. Our analytic identifiability result, which holds in the idealized, noiseless scenario, indicates that costly concentration measurements from blood samples in quantitative PET imaging can be avoided in principle. The connection to noisy measurement data is made via a consistency result, showing that exact reconstruction is maintained in the vanishing noise limit. Numerical experiments with a regularization approach are further carried out to support these analytic results in an application example.

QUANTITATIVE PASSIVE IMAGING BY ITERATIVE HOLOGRAPHY

Björn Müller

Helioseismology uses observations of oscillations at the solar surface in order to study the large-scale flows in the solar interior. The main challenge in this extremely noisy passive imaging problem is the immense size of the input cross-correlation data set as well as the computational costs. Therefore the seismic data is traditionally reduced to a smaller data set of physically interpretable quantities. This reduction looses sensitivity to open questions like the anti-symmetric part of the solar differential rotation. In this talk we discuss the method "iterative holography", which uses the whole seismic data implicitly without storing the cross-correlations at the surface explicitly. Furthermore iterative holography is a converging regularization method, which extends traditional helioseismology to nonlinear problems.

REDUCED ORDER MODEL BASED NONLINEAR WAVEFORM INVERSION

Andreas Tataris

In this talk, we present a new method for solving an inverse scattering problem in the frequency domain using reduced order models (ROMs). We discuss how to obtain the projection of the continuous problem onto the reduced order space spanned by exact solutions using boundary data. Finally, we present a ROM based nonlinear optimisation method aimed to solve the inverse problem, and we compare it to the conventional full waveform inversion.

2 Mixed-Integer Optimization (MIO)

GATE-BASED GROVER ALGORITHM FOR INTEGER LINEAR PROGRAMMING

Christian Fleck
Universität Greifswald

Integer linear programming (ILP) is classified as NP-hard when using classical bit calculators. My goal here is to show a way how an arbitrary ILP can be solved with a quantum computer. I chose a variant of the Grover algorithm, the so-called Grover optimizer. This algorithm is a hybrid algorithm using a classical loop and quantum gates. To use this quantum algorithm it is necessary to transform the original ILP. First, an ILP is transformed into a binary linear program, which in turn is transformed into a quadratic optimization problem without bounds (QUBO). This is done via the penalty method and I found a penalty parameter so that the penalty function is exact. I tested the Grover optimizer implemented in Qiskit to find the minimum of the QUBO resulting from the origin ILP. The original ILP is very small with two variables and one constraint. The implementation of the corresponding QUBO uses 4 qubits to encode the two integer variables and 10 qubits to encode the objective functio

Dynamic scheduling problems

Matthias Gerdts
Universität der Bundeswehr München, Neubiberg

The paper discusses so-called dynamic scheduling problems. Such problems arise in the coordination of interacting agents and aim to find optimal starting times and an optimal starting sequence for the agents taking into account their trajectories. Mathematically this can be modeled as a bi-level optimization problem with a mixed-integer scheduling problem at the upper level and an optimal control problem at the lower level. Herein, both levels are coupled in a non-linear way. We suggest a numerical solution technique, which uses a piecewise linearization technique to transform the bi-level problem into a mixed-integer linear program. Numerical results are presented for coordination scenarios with vehicles at crossings and UAVs aiming to reach prescribed regions. This work is joint work with Viktoriya Nikitina, Sergejs Rogovs, and Jeremy Bertoncini.

CONVEX RELAXATIONS OF PARABOLIC OPTIMAL CONTROL PROBLEMS WITH COMBINATORIAL SWITCHING CONSTRAINTS

Alexandra Grütering
TU Dortmund

We consider optimal control problems for partial differential equations where the controls take binary values but vary over the time horizon, they can thus be seen as dynamic switches. The switching patterns may be subject to combinatorial constraints such as, e.g., an upper bound on the total number of switchings or a lower bound on the time between two switchings. While such combinatorial constraints are often seen as an additional complication that is treated in a heuristic post-processing, the core of our approach is to investigate the convex hull of all feasible switches to define a tight convex relaxation of the control problem. In this talk, we show that the convex relaxation can be built by cutting planes derived from finite-dimensional projections, which can be studied by polyhedral combinatorics, and solved by an outer approximation algorithm. Both the relaxation and the algorithm are independent of any fixed discretization and can thus be formulated in function space.

TIME-DOMAIN DECOMPOSITION FOR MIXED-INTEGER OPTIMAL CONTROL PROBLEMS

Falk Hante
Humboldt-Universität zu Berlin

We consider mixed-integer optimal control problems, whose optimality conditions involve global combinatorial optimization aspects for the corresponding Hamiltonian pointwise in time. We propose a time-domain decomposition, which makes this problem class accessible for mixed-integer programming using parallel-in-time direct discretizations. The approach is based on a decomposition of the optimality system and the interpretation of the resulting subproblems as suitably chosen mixed-integer optimal control problems on subintervals in time. An iterative procedure then ensures continuity of the states at the boundaries of the subintervals via co-state information encoded in virtual controls. We prove convergence of this iterative scheme for discrete-continuous linear-quadratic problems and present numerical results both for linear-quadratic as well as non-linear problems.

ON SOLVING NONLINEAR UNIT COMMITMENT ON GPU

Kibaek Kim
Argonne National Laboratory

We present a decomposition algorithm for solving unit commitment with alternating current optimal power flow (UC-ACOPF), which is formulated to a mixed-integer nonlinear programming problem. We adapt alternating direction method of multiplier (ADMM) that enables the decomposition of UC-ACOPF into grid components, resulting in a large number of small independent optimization subproblems. At each ADMM iteration, the UC is solved by dynamic programming for each generator, and the other components of multi-period ACOPF are solved by a nonlinear optimization solver. Moreover, all the subproblems are solved by employing the massive parallel computing capability of GPU. We demonstrate the numerical performance of the algorithm by using various test instances.

Integer Control Problems with TV-Regularization: Optimality Conditions and Solution Methods

Jonas Marko
BTU Cottbus - Senftenberg

We investigate integer optimal control problems where the goal is to minimize the objective F(u) + βTV(u) with $\beta>0$ such that u(t) is an integer for a.a. t. Here, TV(u) denotes the total variation of u, which penalizes jumps of the control. The contribution F is assumed to be differentiable, e.g. it could realize the tracking of the state given by an ODE or PDE dependent on u. The presence of the TV-term can be used to prove the existence of a minimizer. Moreover, it averts rapid switching between multiple control levels in a short amount of time, which is desirable from an application point of view. Thus, optimization problems of this structure have an abundance of applications.

We show local optimality conditions of first and second order and discuss fast solution methods. Numerical solutions and the corresponding performance of the algorithm will be showcased exemplary on two examples.

SECOND-ORDER PARTIAL OUTER CONVEXIFICATION FOR SWITCHED DYNAMICAL SYSTEMS

Christoph Plate
Otto-von-Guericke Universität Magdeburg

Mixed-integer optimal control problems arise in many practical applications combining nonlinear, dynamic, and combinatorial features. To cope with the resulting complexity, several approaches have been suggested in the past. Some of them rely on solving a reformulated and relaxed control problem, referred to as partial outer convexification. Inspired by an efficient algorithm for switching time optimization by Stellato and coworkers, we developed an algorithmic approach for partial outer convexification. Both approaches are based on linearization and exponential integration to obtain second derivatives. We show the efficiency and applicability of the novel approach by comparing it to the switching time optimization algorithm, by extending the concept and calculations to the treatment of constraints, and by investigating warm-starting of switching time optimization. The new solver facilitates the reliable and fast solution of mixed-integer optimal control problems.

GLOBAL OPTIMIZATION OF MIXED-INTEGER ODE/PDE CONSTRAINED NETWORK PROBLEMS WITH APPLICATION TO GAS TRANSPORT

Stefan Ulbrich
Technische Universität Darmstadt

We consider mixed-integer optimization problems on graphs that involve ODEs or PDEs. We make the structural assumption, that only boundary values of the solution of the ODE or PDE enter the optimization problems, such that the problem can be rewritten by using solution operators of the ODE/PDE. For the ODE-constrained case we propose an adaptive branch-and-bound algorithm for computing global epsilon-optimal solutions of such problems. We combine spatial and variable branching with appropriate adaptively refined discretizations of the differential equations to derive relaxations of the original problem. Convex relaxations are derived for the ODE solution operators using numerical discretization schemes. We show convergence and apply the approach to the example of stationary gas transport. Moreover, we briefly discuss the PDE-case, in particular numerical schemes for the computation of upper and lower bounds of the PDE-solution as well as challenges arising in the PDE context.

3 Model-Order Reduction for Optimization (MOR)

HERMITE KERNEL SURROGATES FOR THE VALUE FUNCTION OF HIGH-DIMENSIONAL NONLINEAR OPTIMAL CONTROL PROBLEMS

Tobias Ehring
University of Stuttgart (D)

Numerical methods for the optimal feedback control of high-dimensional dynamical systems typically suffer from the curse of dimensionality. We present a mesh-free data-based approximation method for the value function, which partially mitigates this problem. The data come from infinite time horizon optimal control systems solved via the Pontryagin's maximum principle, where the infinite horizon is transformed into a finite one, resulting in high quality value function data. The domain of interest is determined by optimal trajectories starting from a predefined set. Here, the most informative initial states are chosen using a greedy selection strategy. The approximation method is based on a greedy Hermite-interpolation scheme, and incorporates context-knowledge. The algorithm is proposed in a matrix-free manner and the convergence of the scheme is proven. Experiments support the effectiveness of the scheme, including a new academic toy model with an explicitly given value function.

REDUCED ORDER MODEL PREDICTIVE CONTROL FOR PARAMETRIZED PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

Saskia Dietze
RWTH Aachen University

Model Predictive Control (MPC) is a well established approach to solve infinite horizon optimal control problems. Since optimization over an infinite time horizon is, in general, infeasible, the method determines a suboptimal feedback control by repeatedly solving finite time optimal control problems.

In this talk, we consider systems governed by parametrized parabolic partial differential equations and employ the reduced basis method (RB) as a low-dimensional surrogate model for the finite time optimal control problem. The reduced order optimal control serves as the feedback control for the MPC of the original large-scale system. Based on rigorous and efficiently computable a posteriori error bounds we are able to guarantee asymptotic stability of the closed-loop system using the RB-MPC approach. We propose an adaptive strategy to choose the optimal horizon length of the finite time optimal control problem and present numerical results to validate our approach.

ADAPTIVE REDUCED BASIS GAUSS NEWTON METHODS FOR INVERSE PROBLEMS

Michael Kartmann
Konstanz University

We are interested in identifying a diffusion parameter q from noisy measurements of a solution u of a parametrized elliptic partial differential equation. Iterative regularization methods typically require a high amount of forward and backward solutions. The authors of [GH16] introduced the Reduced Basis Landweber method (RBL), which uses adaptive RB approximations of the state space to speed up the solution process. Naturally, the extension to higher-order optimization methods is of interest, which is why we modify the Trust Region Reduced Basis Newton (TRRB) method presented in [KMO+21] to a Trust Region Reduced Basis Iteratively Regularized Gauß Newton method (TRRB IRGNM) to account for the solution of the inverse problem. The main challenge here is that the parameter space is infinite-dimensional. Because of that, the key ingredient of our algorithm is to adaptively construct a reduced basis space not only for the state space but also for the parameter space itself.

Accuracy estimates for a Fokker-Planck optimal control problem by a continuous Galerkin approach

Jacob Körner
Julius-Maximilians-Universität of Würzburg (JMU)

A framework for accuracy estimates of optimal control problems with time dependent controls and space-time dependent state is presented. The approach is based on a semidiscrete Galerkin approximation of the state equation and second-order analysis of the optimal control problem. In particular, it is shown that PDE-constrained optimal control problems can be approximated by systems of ODE-constrained ones. Main emphasis is put on the application of this abstract framework to a bilinear, non-convex Fokker-Planck optimal control problem in order to establish accuracy estimates.

SIMPLIFIED NEWTON METHOD FOR POD COMPUTATION IN (INTEGER) OPTIMAL CONTROL

Paul Manns
TU Dortmund University

We interpret the computation of a POD model based on impulse response snapshots as the first step of a simplified Newton iteration on an underlying semilinear evolution equation. These POD models can then be augmented by integrating further impulse response snapshots, which arise from a second step of said simplified Newton iteration. We show that this leads to a high-quality POD model at the cost of an expensive offline phase. We highlight possible applications and extensions in the field of mixed-integer PDE-constrained optimization.

CONSTRAINED OPTIMAL SENSING FOR DIGITAL TWINS

Krithika Manohar
University of Washington - Seattle

We develop a constrained optimization for sensor placement in nuclear digital twins where sensing capability may be severely constrained or limited. These constraints may arise in certain areas of a reactor due to hostile operating conditions, accessibility issues, and physical limitations on sensing capability. Our data-driven method optimizes sensor placement with constraints for full flow field reconstruction, leveraging reduced order models of flow physics. We demonstrate the technique is near optimal using empirical and theoretical validation and provide uncertainty analyses for noisy sensor measurements.

Distributed Control of Partial Differential Equations Using Convolutional Reinforcement Learning

Sebastian Peitz
Universität Paderborn

We present a convolutional framework which significantly reduces the complexity and thus, the computational effort for distributed reinforcement learning control of partial differential equations (PDEs). Exploiting translational invariances, the high-dimensional distributed control problem can be transformed into a multi-agent control problem with many identical agents. Furthermore, using the fact that information is transported with finite velocity in many cases, the dimension of the agents' environment can be drastically reduced using a convolution operation over the state space of the PDE. In this setting, the complexity can be flexibly adjusted via the kernel width or using a stride greater than one. We demonstrate the performance of the proposed framework using several standard PDE examples with increasing complexity, where stabilization is achieved by training a low-dimensional DDPG agent with small training effort.

SPARSITY STRUCTURES FOR THE PERRON-FROBENIUS OPERATOR

Corbinian Schlosser
Inria and ENS Paris

We consider sparse dynamical systems allowing one to consider the system as a family of subsystems interconnected by a graph. Such sparse structures induce a decomposition of the Koopman and Perron-Frobenius operators. Functorial properties of these operators imply that eigenfunctions for the subsystems induce eigenfunctions for the whole system, and invariant measures for the whole system induce invariant measures for the subsystems. We reverse that result for invariant measures under a necessary compatibility condition. We demonstrate, by a numerical example, that exploitation of sparsity improves accuracy for extended dynamic mode decomposition.

MODEL ORDER REDUCTION FOR VARYING BOUNDARY OPTIMAL CONTROL PROBLEMS

Maria Strazzullo
Politecnico di Torino

We introduce varying boundary optimal control (vbOCPs) governed by partial differential equations (PDEs). vbOCPs change the portion of the boundary where the control acts to steer the solution of the PDE to a desired state. This framework may be of interest to energy engineering and geophysics. These fields need fast and reliable simulations for many parameters to deal with time-consuming tasks such as uncertainty quantification or shape design. For this purpose, we rely on reduced order methods (ROMs) to perform accurate simulations in a shorter time. However, vbOCPs might feature transport phenomena and do not benefit from standard ROMs. As a solution, we exploit tailored POD-based approaches, relying on domain transformation or local basis functions, usually employed for challenging tasks such as recovering wave-like phenomena. We compare the novel algorithms with standard POD in terms of accuracy and efficiency on two numerical tests with geometries of increasing complexity.

4 Non-smooth Optimization (NSO)

ANALYSIS OF A VARIATIONAL CONTACT PROBLEM ARISING IN THERMOELASTICITY

Amal Alphonse

We study a model of a thermoforming process involving a membrane and a mould as implicit obstacle problems. Mathematically, the model consists of parabolic and elliptic PDEs coupled to a (quasi-)variational inequality. We study the related stationary (elliptic) model and also the full evolutionary one if time allows. We look at the existence of weak solutions, and by exploring the fine properties of the contact set under non-degenerate data, we give sufficient conditions for the existence of regular solutions. Under certain contraction conditions, we also show a uniqueness result. This is based on a joint paper with Jose-Francisco Rodrigues (Lisbon, Portugal) and Carlos N. Rautenberg (Virginia, USA).

Optimal control of rate-independent systems with non-convex energy

Merlin Andreia
TU Dortmund University

This talk deals with an optimal control problem, where the state variable is given as a parametrized balanced viscosity solution of a rate-independent system with non convex energy. Under certain assumptions on the data one can prove the existence of globally optimal solutions for external loads in $H^1(0,T)$. We investigate the approximability of optimal solutions by viscous regularized problems. The underlying analysis is based on an approximation argument including an additional penalty term, which can be interpreted as a part of the external load ℓ and consequently has to be an element of $H^1(0,T)$. However, it depends on the state and is only continuous in time provided that the optimal state is. In order to weaken this assumption and allow for discontinuous optimal states, one has to deal with external forces in $BV(0,T)$. This however requires to substantially modify the solution concept and leads to concepts that lack physical plausibility as examples demonstrate.

On the nonmonotone FBS algorithm for a class of infinite-dimensional nonsmooth nonconvex problems

Behzad Azmi
University of Konstantz

In this talk, we will discuss the convergence and complexity of the nonmonotone forward-backward splitting (FBS) algorithm for solving a class of nonsmooth composite problems in Hilbert spaces. The objective function is the sum of a Fréchet differentiable (not necessarily convex) function and a lower semicontinuous convex function. These problems appear, for example, frequently in the context of optimization problems governed by nonlinear PDEs with nonsmooth sparsity promoting cost functionals. We will also report on numerical experiments justifying our theoretical findings.

DC REFORMULATION OF CARDINALITY CONSTRAINED PROBLEMS IN FUNCTION SPACES

Bastian Dittrich
Universität Würzburg

We consider cardinality constrained problems in the space of integrable functions. It is shown that this non-convex and discontinuous constraint can be equivalently reformulated by the difference of two convex and continuous functions, namely the L^1-norm and the so called largest-K norm. The convex subdifferential of the largest-K norm is calculated. To solve this problem practically we furthermore provide a finite dimensional exact reformulation for cardinality constraints in the space of piecewise linear functions on arbitrary triangular finite elements. An exemplary problem is solved by applying a DC method to the penalized reformulated problem, for which we also prove an exact penalty result.

Online Optimization for Dynamic Electrical Impedance Tomography

Neil Kristofer Dizon
University of Helsinki

Online optimization generally studies the convergence of optimization methods as more data is introduced into the problem; think of deep learning as more training samples become available. We adapt the idea to dynamic inverse problems that naturally evolve in time. We introduce an improved primal-dual online method specifically suited to these problems, and demonstrate its performance on dynamic monitoring of electrical impedance tomography.

USING SECOND-ORDER INFORMATION IN GRADIENT SAMPLING METHODS FOR NONSMOOTH OPTIMIZATION

Bennet Gebken
Paderborn University

In nonsmooth optimization, solution methods with an order of convergence that is better than linear are difficult to construct and analyze. Part of the problem is the fact that it is not obvious how to define useful and practical higher-order derivative information for an already nonsmooth function. In this talk, as a "second-order derivative", we will consider the so-called second-order eps-jet, which is a set that contains the coefficients of all existing second-order Taylor expansions in an eps-ball around a given point. Based on this concept, we define a model of the nonsmooth objective function as the maximum of these Taylor expansions. While theoretical results on the resulting method are difficult to come by, numerical experiments suggest that in terms of function evaluations, it is often faster than other higher-order methods.

APPROXIMATE OPTIMALITY AND DUALITY FOR NON-SMOOTH SEMI-INFINITE PROGRAMMING PROBLEMS

Shiv Kumar Gupta
Indian institute of Technology Roorkee

The article concentrates on investigating the results for a class of semi-infinite optimization model over arbitrary cone constraints. Firstly, a necessary optimality condition for quasi ϵ- solution of the optimization model is developed using Abadie constraint qualification. Then, the concept of quasiconvexity over cones is introduced and a sufficient optimality condition is proposed using approximate pseudoconvexity and quasiconvexity assumptions. Further, Mond-Weir and Wolfe's dual problems are presented and weak, strong and converse duality results between the semi-infinite optimization model and its dual problems are proved under approximate pseudoconvexity and quasiconvexity assumptions. Moreover, to justify the results, non-trivial numerical illustrations have also been exemplified at suitable places.

On quasidifferentiable mathematical programs with vanishing constraints in Banach spaces

VIVEK LAHA
Banaras Hindu University

The paper deals with mathematical programs with vanishing constraints involving quasidifferentiable functions defined over a real Banach space. We develop Fritz-John and Karush-Kuhn-Tucker type necessary optimality conditions in terms of quasidifferentials of the functions at an optimal point. As these type of optimization problems do not satisfy standard constraint qualifications like Mangasarian-Fromovitz constraint qualification and linear independence constraint qualification, we develop suitable version of some constraint qualifications in terms of quasidifferentials. Some sufficient optimality conditions are also provided. Several future research possibilities are discussed to accommodate a large class of nonconvex and nonsmooth optimization problems.

SPATIALLY SPARSE OPTIMAL CONTROL PROBLEMS

Anna Lentz
Julius-Maximilians-University Würzburg

An optimal control problem with time-dependent controls on a domain $[0, T] \times \Omega$ is considered in fractional Sobolev spaces H^s for $s \in (0, 1)$. To obtain spatially sparse controls, the objective function contains the sparsity promoting L^p-pseudo norm for $0 < p < 1$, which makes the problem non-convex and non-smooth. In order to avoid computing the fractional Laplacian on $[0, T] \times \Omega$, we introduce an auxiliary function $w \in H^s(\Omega)$ such that

$$|u(t, x)| \leq w(x) \qquad \text{for almost all } (t, x) \in [0, T] \times \Omega,$$

so w is an upper bound for the control $u \in L^2([0, T] \times \Omega)$. Optimality conditions and regularity results for this problem are discussed. This is done by smoothing the L^p-pseudo norm and by penalizing the inequality constraint regarding u and w.

MINIMIZING THE KREISS CONSTANT

Dominikus Noll
Université de Toulouse

The Kreiss constant K(A) of a stable matrix A attains its global minimum K(A)=1 when exp(At) is a contraction in the spectral norm, and in particular, for normal matrices A. K(A) is therefore often understood as a 'measure of normality' of A. The larger K(A) > 1, the less normal A is.

In a nonlinear dynamical system x' = Ax + f(x), locally stable at 0, non-normal A may lead to large transients, and this may trigger instability, as trajectories have then a higher tendency to leave the neighborhood of local stability of 0.

In control the non-linear closed-loop x' = A(k)x + f(x) still depends on our choice of the controller k, and this gives us freedom to optimize A(k). We might tune the controller k such that A(k) becomes 'more normal', e.g., by minimizing its Kreiss constant K(A(k)). This type of optimization is non-smooth.

INEXACT PROXIMAL NEWTON METHODS IN HILBERT SPACES

Anton Schiela
Universität Bayreuth

We consider proximal Newton methods in Hilbert spaces for the solution of nonsmooth variational problems. In contrast to classical proximal gradient methods on R^n these methods enjoy fast local convergence and thus need only few iterations until convergence. However, the subproblems are typically harder, and we apply an inner nonsmooth multigrid solver (TNNMG) for their efficient inexact solution. We discuss inexactness criteria and their relation to global and fast local convergence of our algorithm. Finally we present some applications and numerical examples.

OPTIMIZATION OF PIECEWISE-SMOOTH SHAPES WITH VARIATIONAL INEQUALITY CONSTRAINTS BY A HADAMARD SEMIDERIVATIVE APPROACH

Tim Suchan
Helmut Schmidt University / University of the Federal Armed Forces Hamburg

Theoretical analysis of shape optimization problems constrained by variational inequalities (VI) are a challenging field of research due to the combination of the differential-geometric background of shape optimization problems combined with the non-differentiability in the classical sense due to the VI, which usually requires a regularization technique to handle the non-differentiability. Recent progress in the field of Riemannian shape manifolds has enabled the development of kinks during the optimization process. Further, a Hadamard semiderivative approach provides the possibility to use the unregularized form of the VI during the optimization. In this talk, we combine these two fields and show numerical results for a VI-constrained shape optimization problem while allowing piecewise-smooth shapes.

SEMISMOOTHNESS OF THE SOLUTION OPERATOR OF THE OBSTACLE PROBLEM WITH APPLICATIONS IN OPTIMAL CONTROL

Gerd Wachsmuth
BTU Cottbus - Senftenberg

We address the solution mapping $f \mapsto u$ of the classical obstacle problem

$$u \in K, \quad \langle -\Delta u - f, v - u \rangle_{H_0^1(\Omega)} \geq 0 \ \forall v \in K$$

in which the closed, convex set $K \subset H_0^1(\Omega)$ is given by unilateral or bilateral constraints. It is shown that this solution operator is semismooth. As an application, we consider the optimal control problem

$$\begin{aligned} \text{Minimize} \quad & \&\frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\alpha}{2}\|\nabla u\|^2_{L^2(\Omega)} \\ \text{such that} \quad & \&-\Delta y + y = u \text{ in } \Omega,\ \partial_n y = 0 \text{ on } \partial\Omega \\ & \&u \in K. \end{aligned}$$

The main feature of this problem is the $H_0^1(\Omega)$-regularization of the control. The necessary optimality condition of the control problem contains the obstacle problem. Due to the semismoothness, we can apply the semismooth Newton method.

ON THE SOLUTION OF CONSTRAINED NONSMOOTH OPTIMIZATION PROBLEMS

Andrea Walther
Humboldt-Universität zu Berlin

The solution of nonsmooth constrained optimization problems is required for numerous applications including also various training tasks from machine learning. The class of abs-smooth functions covers a large fraction of corresponding nonsmooth target functions. We present an optimization method based on the Frank-Wolfe approach for this function class, where the piecewise linear inner problem is solved by the active signature method. The convergence of the method including also an analysis of the convergence rate is presented. Finally, we discuss implementation options and first numerical results.

5 Optimal and Feedback Control of PDEs (PDE)

ACOUSTIC FULL-WAVEFORM INVERSION VIA OPTIMAL CONTROL

Luis Ammann
University of Duisburg-Essen

Full-waveform inversion (FWI) is a recent technique in seismic to- mography to reconstruct physical parameters sampled by waves. Com- pared with other methods relying only on partial waveform information such as travel times or phase velocities, FWI exploits the entire wave- form content. In this talk, we discuss an optimal control method for acoustic FWI. The aim is to reconstruct the speed wave parameter en- tering the hyperbolic PDE model in the coefficient of the second-order time derivative of the acoustic pressure. For the given optimization problem, we present necessary first-order optimality conditions based on adjoint techniques where the adjoint state has only low regular- ity properties. This is particularly favorable since then no Sobolev smoothing effect occurs in the optimal solution. Further, under spe- cific regularity and compatibility assumptions, we present second-order sufficient optimality conditions and discuss the essential ideas of the non-standard proof.

ON THE STABILIZATION OF A KINETIC MODEL BY FEEDBACK-LIKE CONTROL FIELDS IN A MONTE CARLO FRAMEWORK

Jan Bartsch
University of Konstanz

The linear Boltzmann equation is frequently used to describe the evolution of multiple particle systems including collisions. In this talk, we investigate the construction of feedback-like control fields for a kinetic model in phase space using a linear Boltzmann equation with a collision term. The purpose of these controls is to drive an initial density of randomly distributed particles in the phase space to reach a desired cyclic trajectory in the phase space and follow it in a stable way. To achieve this, we formulate an ensemble optimal control problem governed by the kinetic model in a way that is amenable to a Monte Carlo approach. The proposed formulation allows us to define a one-shot solution procedure consisting in a backward solve of an augmented adjoint kinetic model. Results of numerical experiments demonstrate the effectiveness of our proposed control strategy.

Strong stationarity for a highly nonsmooth optimization problem with control constraints

Livia Betz
Universität Würzburg

In this talk we present a control constrained optimization problem governed by a nonsmooth elliptic PDE in the presence of a non-differentiable objective. The nonsmooth non-linearity in the state equation is locally Lipschitz continuous and directionally differentiable, while one of the nonsmooth terms appearing in the objective is convex. Since these mappings are not necessarily Gâteaux-differentiable, the application of standard adjoint calculus is excluded. Based on their limited differentiability properties, we derive a strong stationary optimality system, i.e., an optimality system which is equivalent to the purely primal optimality condition saying that the directional derivative of the reduced objective in feasible directions is nonnegative.

THE SEQUENTIAL QUADRATIC HAMILTONIAN METHOD

Alfio Borzì
Würzburg University

The sequential quadratic hamiltonian (SQH) method is a novel numerical optimization procedure for solving optimal control problems governed by differential models. It is based on the characterization of optimal controls in the framework of the Pontryagin maximum principle (PMP). It represents the most recent development of the so-called successive approximations schemes. In this talk, results of analysis and application of the SQH method to different classes of PDE optimal control problems are presented.

Approximations of Operator Lyapunov Equations with Applications to Nonlinear Feedback Control

Tobias Breiten
Technische Universität Berlin

Computing the Lyapunov function of a system plays a crucial role in optimal feedback control, for example when the policy iteration is used. This talk will focus on the Lyapunov function of a nonlinear autonomous finite-dimensional dynamical system which will be rewritten as an infinite-dimensional linear system using the Koopman operator. Since this infinite-dimensional system has the structure of a weak-* continuous semigroup in a specially weighted Lp-space one can establish a connection between the solution of an operator Lyapunov equation and the desired Lyapunov function. It will be shown that the solution to this operator equation attains a rapid eigenvalue decay, which justifies finite rank approximations with numerical methods. The usefulness for numerical computations will also be demonstrated with two short examples.

A damped elastodynamics system under the global injectivity condition: Modeling defibrillation via a hybrid optimal control problem

Sebastien Court
Universität Innsbruck

We aim at modeling mathematically certain mechanical aspects of defibrillation. The time deformation of the heart tissue is modeled with the elastodynamics equations dealing with the displacement field as main unknown. These equations are coupled with a pressure whose variations characterize the defibrillation process. The pressure variable corresponds to a Lagrange multiplier associated with the so-called global injectivity condition. We derive the corresponding coupled system with Neumann boundary conditions, and develop a general hybrid optimal control approach that covers in particular the maximization of the variations of this pressure, and also the time the maximum is reached. For mathematical convenience a damping term is added, and mathematical analysis based on the L^p-parabolic maximal regularity is provided for the state equations and the rigorous derivation of optimality conditions. Numerical simulations exploit these optimality conditions and illustrate the approach.

POINTWISE ERROR ESTIMATES FOR NUMERICAL SOLUTIONS TO LINEAR-QUADRATIC OPTIMAL CONTROL PROBLEMS

Sebastian Hofmann
Julian-Maximilians-Universität Würzburg

A technique for estimating the global errors of approximate solutions to linear-quadratic optimal control problems is presented. For this purpose, an auxiliary optimal control problem is formulated that provides, with its unique solution, a continuous representation of the global errors of the discrete control and the corresponding state. The resulting error functions are characterized as the unique solutions to an optimality system which, after reformulation, can be solved utilizing ideas of defect correction and defect control. It is validated numerically that with this approach, reliable pointwise estimates of the global error of numerical solutions to linear-quadratic optimal control problems can be generated.

CONTROL OF SLIDING DROPLETS USING THE CONTACT ANGLE

Christian Kahle
Universität Koblenz

We present results on optimal control of sliding droplets. Here the contact angle between droplet and solid serves as a control variable. The fluid is modeled by a thermodynamically consistent diffuse interface model with a suitable contact line model.

In earlier work [H. Bonart, C. Kahle, J.-U. Repke, JCP 399 (2019)] we compared different time discretization schemes for this model that mimics the energy behaviour of the continuous model. We now employ a particular scheme to derive existence of optimal controls for a time discrete optimal control problem and also first order neccessary conditions.

As controls we consider finite dimensional controls for the contact angle distribution. We test our approach by driving a droplet up an inclined plate.

ON THE IDENTIFICATION AND OPTIMIZATION OF NONSMOOTH SUPERPOSITION OPERATORS IN SEMILINEAR ELLIPTIC PDES

Julia Kowalczyk
TUM

We want to identify a non-linear superposition operator that occurs in the semilinear PDE constraint of an optimal control problem. Extending the currently existing analysis, the superposition operator is not assumed to be an element of $C^1(\mathbb{R})$ but of $H^1_{\text{loc}}(\mathbb{R})$. In this way, we derive the theoretical foundations for recent neural network discretisation approaches in which the activation function is chosen to be piecewise affine and continuous. A particular difficulty in the $H^1_{\text{loc}}(\mathbb{R})$-setting is the derivation of first-order necessary conditions due to the a priori non-smooth control-to-state mapping. However, under certain assumptions, its Fréchet-differentiability can be established. The optimality system then enables us to formulate and prove the convergence of a projected gradient descent method in the infinite-dimensional as well as finite-dimensional space. This talk is based on joint work with Constantin Christof.

CONTROL PARAMETRIZATION FOR OPTIMAL CONTROL PROBLEMS GOVERNED BY PARTIAL DIFFERENTIAL EQUATIONS

Simone Kulka
Universität Greifswald

The presentation is about how the approach of control parametrization (which is well studied in the ordinary case) can be generalized to problems governed by partial differential equations. In order to use nonlinear optimization-techniques, one is interested in the gradients of the obtained optimization problem with respect to the parameters. Unfortunately, these gradients are non-smooth in general but perhaps there is an analytical way to compute these. The presentation is illustrated by a simple example. So far, only linear partial differential equations with piecewise constant control elements on rectangular domains have been considered. This could be the first step to successfully extend the approach to nonlinear PDEs and other types of basis functions and switching structures. The method could be particularly useful in detecting switching hyperplanes, but also allow more flexible control functions for smoother problems.

Optimal Feedback Control of Nonlinear Partial Differential Equations: Taming the curse of dimensionality

Karl Kunisch
Karl-Franzens University of Graz

Optimal feedback controls for nonlinear systems are characterized by the solutions to a Hamilton Jacobi Bellmann (HJB) equation. In the deterministic case, this is a first order hyperbolic equation. Its dimension is that of the statespace of the nonlinear system. Thus solving the HJB equation is a formidable task and one is confronted with a curse of dimensionality.

In practice, optimal feedback controls are frequently based on linearisation and subsequent treatment by efficient Riccati solvers. This can be effective, but it is local procedure, and it may fail or lead to erroneous results.

I give a brief survey of current solution strategies to partially cope with this challenging problem.

Subsequently I describe two approaches in some detail. The first one is a data driven technique, which approximates the solution to the HJB equation and its gradient from an ensemble of open loop solves.

The second technique circumvents the direct solution of the HJB equation. It is based on a succinctly chosen learning ansatz, with subsequent approximation of the feedback gains by neural networks or polynomial basis functions.

This work relies on collaborations with B.Azmi (Uni-Konstanz), D.Kalise(Imperial College), D. Vasquez-Varas (Ricam Austrian Acad. Sciences) , and D.Walter (Humbold Universität Berlin).

REGULARIZATION AND OUTER APPROXIMATION FOR OPTIMAL CONTROL PROBLEMS WITH CONTROLS IN BV

Christian Meyer
TU Dortmund

We consider a convex elliptic optimal control problem with a constraint on the TV-seminorm of the control. We apply a dual regularization of the TV seminorm and solve the resulting optimization problems with an outer approximation algorithm. We prove the convergence of the algorithm to the global optimal solutions, which in turn converge to the optimal solution to the original problem as the regularization parameter tends to zero. The theoretical findings are confirmed by numerical experiments.

GLOBAL STABILIZATION OF THE SCHLOEGL PARABOLIC EQUATION UNDER CONTROL CONSTRAINTS

Sergio S. Rodrigues
Johann Radon Institute for Computational and Applied Mathematics, Austrian Academy of Sciences

An explicit saturated feedback control input is presented which is able to stabilize the nonlinear Schloegl parabolic equation. The performance of this feedback is then compared to that of an optimal feedback control input minimizing a classical quadratic cost functional on the infinite time-horizon. The computation of an approximation of the latter is done by following a receding horizon strategy based on analogue optimal control problems defined on overlapping finite time-horizons. The approximation of an infinite time-horizon solution by finite time-horizon ones is discussed.

A VARIATIONAL CALCULUS FOR OPTIMAL CONTROL OF NETWORKS OF SCALAR CONSERVATION OR BALANCE LAWS

Marcel Steinhardt
Technische Universität Darmstadt

Networks of scalar conservation laws provide models for vehicular traffic flow, supply chains or transmission of data. Such networks consist of initial boundary value problems (IBVPs) of scalar conservation laws on every edge coupled by node conditions. For their optimal control a variational calculus is desirable that implies differentiability of objective functionals w.r.t. controls. In the last decade research on IBVPs successfully introduced a variational calculus which implies differentiability of objective functionals of tracking type and yields an adjoint based gradient representation. This talk presents recent progress in an extension of these results to networks of scalar conservation laws. Regarding node conditions we introduce a framework for their representation compatible with the known approach on single edges allowing to extend results such as continuous Fréchet differentiability and the adjoint based gradient representation of particular functionals on the network.

INSTANTANEOUS CONTROL OF INTERACTING PARTICLE SYSTEMS

Claudia Totzeck
University of Wuppertal, Faculty of Mathematics and Natural Sciences

In this talk we discuss the port-Hamiltonian structure of interacting particle systems with consensus and repulsion/attraction dynamics. Begining with the microscopic description we see that the structure is preserved while passing to the mean-field limit. We will see how long-time behaviour can be characterised using the port-Hamiltonian framework. Moreover, we discuss control approaches by coupling different species. The theoretical results are underlined with numerical examples.

RANDOM BATCH METHODS FOR OPTIMAL CONTROL OF 1D HYPERBOLIC SYSTEMS

Yue Wang
Friedrich - Alexander - Universität Erlangen - Nürnberg

Optimal control for networks of hyperbolic systems is important in many applications, such as gas networks, flexible multi-body systems, and water networks. However, solving such optimal control problems can be computationally demanding when the network is large or contains loops. In this talk, we present new convergence results for the simulation and optimal control of 1D hyperbolic systems by the Random Batch Method (RBM). The RBM is a recently proposed randomized operator-splitting technique inspired by the successes of stochastic algorithms in machine learning [Shi Jin, Lei Li, Jian-Guo Liu, J. of Comp. Phys. , 2020]. The results in this talk are the first extension of the analysis for finite-dimensional optimal control problems in [Veldman, Zuazua, Numer. Math., 2022] to hyperbolic partial differential equations. A numerical example for a network with loops shows that the proposed method reduces the computational cost significantly.

6 Optimization and Machine Learning (ML)

A MULTIOBJECTIVE CONTINUATION METHOD TO COMPUTE THE REGULARIZATION PATH OF DEEP NEURAL NETWORKS

Augustina Chidinma Amakor
Universität Paderborn

Sparsity is a highly desired feature in deep neural networks (DNNs) as it ensures numerical efficiency, improves interpretability of models and robustness. In machine learning approaches based on linear models, there exists a connecting path between the sparsest solution in terms of the L1 norm and the non-regularized solution, called the regularization path. Recently, there was an attempt to extend the concept of regularization paths to DNNs by means of treating the non-regularized loss and sparsity as two conflicting criteria and solving the resulting multiobjective optimization problem. Due to the non-smoothness of the L1 norm, this approach is not very efficient from a computational perspective. Hence, we present an algorithm for the approximation of the entire Pareto front for the above-mentioned objectives in a very efficient manner. We prove convergence in the deterministic case and present numerical examples using both deterministic and stochastic gradients.

LEARNING UNDER ROBUSTNESS CONSTRAINTS

Luiz F. O. Chamon
Universität Stuttgart

While adversarial training methods has been shown to mitigate the fragility of deep learning models to input perturbations, they are increasingly application-dependent, heuristic in nature, and suffer from trade-offs between nominal performance and robustness. Moreover, the problem of finding worst-case perturbations is non-convex and underparameterized, engendering a non-favorable optimization landscape. Thus, there is a gap between the theory and practice of adversarial training, particularly as to when and why it works. In this talk, we take a constrained learning approach to these questions leveraging semi-infinite optimization and non-convex duality to show that adversarial training is equivalent to a statistical problem over a closed-form perturbation distribution. Using a smooth approximation of this distribution and a hybrid of Langevin Monte Carlo-stochastic optimization algorithm, we obtain state-of-the-art results on MNIST and CIFAR-10 benchmarks.

OPTIMAL APPROXIMATION FROM POINT VALUES

Matthieu Dolbeault
RWTH Aachen

In this talk, we investigate sampling strategies for approximation of functions by weighted least-squares. Although a quasilinear sampling budget can already be achieved by iid random draws according to the adapted density, further reductions of the needed number of points are possible, using the solution to the Kadison-Singer problem. However this involves a subsampling step, which is not algorithmically tractable. We show how greedy sampling methods can circumvent this defect, while attaining optimal sample sizes.

PHYSICS-INFORMED NEURAL NETWORKS FOR OPTIMAL CONTROL PROBLEMS

Evelyn Herberg
Ruprecht-Karls-University Heidelberg

We consider an optimal control problem in Banach spaces and its optimality system. To solve the optimal control problem, we construct a learning problem utilizing the Physics-Informed Neural Network (PINN) approach. Essentially, the equations from the optimality system evaluated at collocation points are inserted as penalty terms in the PINN loss function. In this way the network is informed by the optimality system. We present numerical examples to illustrate our results.

MULTI-OBJECTIVE OPTIMIZATION FOR DEEP NEURAL NETWORK TRAINING USING WEIGHTED CHEBYSHEV SCALARIZATION

Sedjro Salomon Hotegni
Paderborn University

Different conflicting criteria arise naturally in deep learning. These can address different main tasks (i.e., in the setting of multi-task learning), but also main and secondary tasks such as loss minimization versus sparsity. The usual approach is a simple weighting of the criteria, which formally only works in the convex setting. In this presentation, we present a Multi-Objective Optimization algorithm using Weighted Chebyshev (WC) scalarization in conjunction with the Augmented Lagrangian (AL) method for training Deep Neural Networks with respect to two tasks. By employing WC scalarization, the algorithm can identify all optimal solutions to the original MOO problem while reducing its complexity to a sequence of single-objective problems. The simplified problem is then solved using the AL method, enabling the use of popular optimization techniques such as Adam and SGD while effectively handling constraints. We demonstrate the performance on a novel, challenging multi-task problem.

A RECURSIVE MULTILEVEL ALGORITHM FOR DEEP LEARNING

Isabel Jacob
Technische Universität Darmstadt

With more challenging use cases and the increasing complexity of modern neural networks, the search for more efficient learning algorithms becomes more critical. Multilevel methods, traditionally applied in solving differential equations using a hierarchy of discretisations, offer the potential to reduce computational effort.

In this talk, we combine both concepts and introduce a multilevel stochastic gradient descent algorithm that accelerates learning by a multilevel strategy. A gradient correction term is needed to establish first-order consistency. We discuss further conditions to ensure the convergence of the method.

To demonstrate the usefulness of our approach, we apply it to residual neural networks in image recognition using image resolution to establish a hierarchy. We construct neural networks of decreasing number of variables and corresponding prolongation and restriction operators. Numerical results are presented.

ADAPTIVE SGD WITH POLYAK STEPSIZE: ROBUST CONVERGENCE AND VARIANCE REDUCTION

Xiaowen Jiang
CISPA - Helmholtz-Zentrum für Informationssicherheit gGmbH

The recently proposed stochastic Polyak stepsize (SPS) for SGD has shown remarkable effectiveness when training over-parameterized models. However, in non-interpolation settings, it only guarantees convergence to a neighborhood of a solution which may result in a worse output than the initial guess. In this work, we make two contributions: Firstly, we propose a new variant of SPS, called AdaSPS, which guarantees convergence in non-interpolation settings and maintain sub-linear and linear convergence rates for convex and strongly convex functions when training over-parameterized models. AdaSPS requires only a lower bound of the optimal function value as input. Secondly, we equip AdaSPS with a novel variance reduction technique and obtain an algorithm that is more efficient than AdaSPS when optimizing non-interpolated convex functions. Moreover, our result matches the fast rates of AdaSVRG and SARAH but removes the inner-outer-loop structure, which is easier to implement and analyze.

STOCHASTIC MODIFIED FLOWS, MEAN-FIELD LIMITS AND DYNAMICS OF STOCHASTIC GRADIENT DESCENT

Sebastian Kassing
University of Bielefeld

We propose new limiting dynamics for stochastic gradient descent in the small learning rate regime called stochastic modified flows. These SDEs are driven by a cylindrical Brownian motion and improve the so-called stochastic modified equa- tions by having regular diffusion coefficients and by matching the multi-point statis- tics. As a second contribution, we introduce distribution dependent stochastic modi- fied flows which we prove to describe the fluctuating limiting dynamics of stochastic gradient descent in the small learning rate - infinite width scaling regime.

CONVERGENCE OF GRADIENT DESCENT WITH LINEARLY CORRELATED NOISE AND APPLICATIONS TO DIFFERENTIALLY PRIVATE LEARNING

Anastasiia Koloskova
EPFL

We study stochastic optimization with linearly correlated noise. Our study is motivated by recent methods for optimization with differential privacy (DP), such as DP-FTRL, which inject noise via matrix factorization mechanisms. We propose an optimization problem that distils key facets of these DP methods and that involves perturbing gradients by linearly correlated noise. We derive improved convergence rates for gradient descent in this framework for convex and non-convex loss functions. Our theoretical analysis is novel and might be of independent interest. We use these convergence rates to develop new, effective matrix factorizations for differentially private optimization, and highlight the benefits of these factorizations theoretically and empirically.

Mirror Descent with Relative Smoothness in Measure Spaces, with application to Sinkhorn and EM

Anna Korba
CREST-ENSAE

Many problems in machine learning can be formulated as optimizing a convex functional over a vector space of measures. This paper studies the convergence of the mirror descent algorithm in this infinite-dimensional setting. Defining Bregman divergences through directional derivatives, we derive the convergence of the scheme for relatively smooth and convex pairs of functionals. Such assumptions allow to handle non-smooth functionals such as the Kullback–Leibler (KL) divergence. Applying our result to joint distributions and KL, we show that Sinkhorn's primal iterations for entropic optimal transport in the continuous setting correspond to a mirror descent, and we obtain a new proof of its (sub)linear convergence. We also show that Expectation Maximization (EM) can always formally be written as a mirror descent. When optimizing only on the latent distribution while fixing the mixtures parameters – which corresponds to the Richardson–Lucy deconvolution scheme in signal processing – we derive sublinear rates of convergence.

ADAPTIVE STEP SIZE CONTROL FOR STOCHASTIC OPTIMIZATION

Frederik Köhne
Universität Bayreuth

The training process of machine learning models usually requires several hyperparameters to be set in advance. The lack of clear guidelines on how to choose such hyperparameters optimally leads to computationally expensive testing of several hyperparameter-settings. A prominent example of such a hyperparameter is the step size (or learning rate) for a stochastic optimization algorithm like Stochastic Gradient Descent (SGD). In this talk, we show that in a special setting it is possible to derive the step size adaptively from quantities, that can be computed relatively cheep during the run of the algorithm. This leads to a version of SGD with adaptive step size control. It is shown, that step size selection and convergence properties can benefit from preconditioning, when the algorithm is applied to stochastic quadratic problems. We demonstrate, that this algorithm performs well even in problem domains not covered by theory, e.g. classic image classification tasks.

On Semi-global Optimal Control and its Application to Machine Learning

Mathias Oster
RWTH Aachen

We seek to learn a function by deep neural networks. An abstract optimal control problem with measure-valued controls provides an interesting mathematical framework to analyse the expressivity and optimization of such networks from a continuous point of view. This control problem can be seen as an infinite deep neural network where the last layer is of a special form. We exploit the ideas of Barron spaces as continuous interpretation of infinite wide shallow networks and neural ODEs as infinite deep residual network architectures. We show the existence of minimizers to the optimal control problem by using Prokhorov's theorem on tight measures and some regularity assumptions on the activation function. Secondly, we analyse corresponding gradient flows in the space of probability measures endowed with the Wasserstein metric.

A DIMENSIONALITY REDUCTION TECHNIQUE FOR GLOBAL OPTIMIZATION VIA RANDOM SUBSPACE EMBEDDINGS

Adilet Otemissov
Nazarbayev University

In an attempt to improve the scalability of global optimization problems, we propose a random algorithmic framework, which tackles the global optimization problem by repeatedly solving subproblems with the domains restricted to be along randomly embedded low-dimensional subspaces. We analyse the convergence of the proposed framework for Lipschitz continuous functions using tools from conic integral geometry/random matrix theory. We then particularise the framework and analysis for functions with low effective dimensionality, which vary only over the (small-dimensional) effective subspace. We show that for these functions the convergence could be exponentially better and that, under certain assumptions, it does not depend on the ambient dimension. Our numerical experiments illustrate: 1) the improved scalability when coupled with state-of-the-art global and local optimization solvers 2) the ability to estimate the unknown effective dimension of the objective function.

ACCELERATED RATES BETWEEN STOCHASTIC AND ADVERSARIAL ONLINE CONVEX OPTIMIZATION

Sarah Sachs
University of Amsterdam

Stochastic and adversarial data are two widely studied settings in online learning. But many optimization tasks are neither i.i.d. nor adversarial, which makes it of fundamental interest to get a better theoretical understanding of the world between these extremes. We establish novel regret bounds for online convex optimization in a setting that interpolates between stochastic i.i.d. and fully adversarial losses. By exploiting the smoothness of the expected losses, these bounds replace dependence on the maximum gradient length by the variance of the gradients. In addition, they weaken the i.i.d. assumption by allowing, e.g., adversarially poisoned rounds. In the fully i.i.d. case, our regret bounds match the rates one would expect from stochastic acceleration results. Furthermore, the optimal stochastically accelerated rates are recovered via online-to-batch conversion. In the fully adversarial case, our bounds gracefully deteriorate to match the minimax regret.

INEXACT ALGORITHMS FOR BILEVEL LEARNING

Mohammad Sadegh Salehi
University of Bath

Hyperparameter estimation is crucial in Machine Learning. When considering tasks in imaging inverse problems modelled in the variational regularization approach, manually configuring regularization parameters is challenging, even with a single parameter. The difficulty is further amplified when employing neural network-based regularizers, which can involve millions of hyperparameters.

We address this challenge by learning the hyperparameters using bilevel learning, leading to a nested optimisation problem. Due to the use of numerical solvers, the exact gradient with respect to the hyperparameters cannot be computed, necessitating the use of methods relying on approximate lower-level solutions and gradients.

We present an analysis of hypergradient estimation and a novel inexact line search framework which determines the required accuracy adaptively. We validate the empirical performance of our framework and compare it numerically with different methods on an image denoising problem

CURSE-OF-DIMENSIONALITY-FREE APPROXIMATION OF OPTIMAL VALUE FUNCTIONS WITH NEURAL NETWORKS

Mario Sperl
Universität Bayreuth

In this talk, we consider an optimal control problem with interconnected dynamics and discuss the ability of neural networks to compute curse-of-dimensionality-free approximations of the corresponding optimal value function. To this end, we build on the fact that deep neural networks are capable of approximating separable functions with a number of neurons that grows only polynomially in the state dimension. We represent the interconnected control system as a graph of subsystems and assume a decaying sensitivity property of the subsystems with their distance in the graph. It is then shown how this assumption can be used to construct a separable approximation of the optimal value function. Further, a neural network architecture for the corresponding computation is presented.

FROM GRADIENT FLOW FORCE-BALANCE TO DISTRIBUTIONALLY ROBUST LEARNING

Jia-Jie Zhu
Weierstrass Institute for Applied Analysis and Stochastics, Berlin

Motivated by the force-balance formulation of PDE gradient flows, I will demonstrate using the optimality condition in the dual space to solve distributionally robust optimization (DRO) problems, which seek learning and optimization solutions under distribution shifts.

One of the recent breakthroughs of DRO is the adoption of the Wasserstein geometry. While the literature has exploded, it is restricted to the pure transport regime, i.e., no creation or destruction of mass. Its practical effect is also similar to existing regularization techniques already used by machine learners. In addition, many state-of-the-art Wasserstein DRO algorithms place severe limitations on the learning models and scalability. With those in mind, I will introduce applications of the duality principle beyond the Wasserstein DRO using unbalanced optimal transport and kernel geometry.

7 Optimization in Applications (APP)

Towards explainable and privacy-preserving machine learning

Anastasia Borovykh
Imperial College London

Machine learning, and specifically deep neural networks, are increasingly deployed in the real world. Despite the satisfactory performance achieved in practical applications, these models are generally difficult to analyze and their performance is not always fully understood. This impacts their deployment as it influences two critical real-world challenges: generalisation - guaranteeing good performance of the model in unseen scenarios and privacy - ensuring the trained model does not give away sensitive information about the datasets it was trained on. In this talk we will first go into more detail on the challenges associated with generalisation and privacy. We will then discuss several recent advancements in defining robust and privacy-preserving optimization algorithms, focusing on interacting particle optimization and Langevin dynamics with anisotropic noise.

THEORETICAL AND NUMERICAL COMPARISON OF FIRST-ORDER ALGORITHMS FOR SMOOTH CONVEX OPTIMIZATION

Luis Briceño-Arias
Universidad Técnica Federico Santa María

In this talk, we provide a theoretical and numerical comparison of classical first-order splitting methods for solving smooth convex optimization problems arising, e.g., in image and signal processing problems. Theoretically, we compare convergence rates of gradient descent, forward-backward, Peaceman-Rachford, and Douglas-Rachford algorithms for minimizing the sum of two smooth convex functions when one is strongly convex. In several instances, we obtain improved rates with respect to the literature by exploiting the structure of our problems. Moreover, we indicate which algorithm has the lowest convergence rate depending on the strong convexity parameter and the Lipschitz constant of the gradients. We verify our theoretical results numerically by implementing and comparing previous algorithms in well-established signal and image inverse problems involving sparsity.

A NOVEL HOMOTOPY APPROACH FOR THE RELIABLE DETERMINATION OF MODEL PARAMETERS FOR CHROMATOGRAPHIC PROCESSES

Dominik H. Cebulla
TU Braunschweig

Column liquid chromatography plays an important role in the downstream processing of biopharmaceuticals, where the goal is to capture and purify a target protein from a mixture. Our goal is to optimize a real-world ion exchange chromatography process using a model-based approach.

We particularly focus on the model calibration step, as convergence of the underlying optimization procedure for parameter estimation is hard to achieve in practice, partly due to the lack of sufficient data and good initial guesses for the parameters. To overcome this undesired behavior, we propose a novel homotopy approach based on exponentially modified Gaussian functions, the latter being known to describe chromatography data well. We will see that this approach is capable of reliably determining suitable model parameters.

We conclude our talk with an outlook on the predictability of the calibrated model by comparing real-world data from the optimized chromatographic process with our model response.

Optimization of an Energy Storage Process Using Iron as Energy Carrier

Elisa Corbean
TU Darmstadt

This work presents an optimal design problem for an energy storage process via thermochemical reduction of iron oxides. The material properties of iron allow the efficient storage and transport of renewable energies and it could thus play an important role for the energy transition.

A mathematical model is developed and optimized with the goal of finding the optimal component dimensioning and operation strategy of the considered process. It comprises renewable energy systems, an electrolyzer for the production of green hydrogen and an innovative reduction plant. The resulting optimization problem includes ODE constraints for modelling the reduction reaction, nonlinear constraints describing the underlying physical processes of the components and an economic objective function. The presented model allows for location specific designs of the storage process depending on the availability of renewable energy sources.

Reinforcement Learning for Space Debris Removal

Simon Gottschalk
Universität der Bundeswehr München

In this presentation, we focus on a docking maneuver in space to remove space debris in the orbit. In addition to complex dynamical systems, this problem class suffers from non-convex pure state constraints for collision avoidance. Thereby, classical optimal control methods reach their limits. We discuss how Reinforcement Learning (RL), which is able to treat a high number of complicated state constraints, can support classical strategies in a hierarchical way to find a solution. In the end, we will have an approach, which benefits from RL as well as from classical strategies. Thereby, RL takes care of the collision avoidance while the classical strategy steers the actual satellite in a smooth manner.

The overall strategy is tested on a computer-aided simulation as well as on a laboratory test scenario, where robotic arms simulate the behavior of docking satellites.

PDE-CONSTRAINED OPTIMIZATION PROBLEMS IN THE FIELD OF SHALLOW GEOTHERMAL ENERGY

Smajil Halilovic
Technical University of Munich (TUM)

Shallow geothermal systems (SGSs) play an important role in decarbonizing the heating and cooling sector, making them a crucial technology for the energy transition. Optimization of SGSs falls into the area of PDE-constrained optimization since the underlying physical phenomena, i.e. groundwater flow and subsurface heat transport, are described by PDEs. In this talk, we focus on two specific cases of such problems: design optimization of open-loop SGSs, which involves optimal well placement, and optimal control of a closed-loop SGS. To address these optimization problems, we propose adjoint-based optimization approaches that use the Firedrake framework for the forward simulation model and the Dolfin adjoint tool for automatically generating discrete adjoint PDEs. In addition, we highlight the underlying challenges associated with these optimization problems, including non-convexity and non-smoothness, and discuss possible directions for future improvements.

TRANSATLANTIC FLIGHT PLANNING WITH DYNAMIC PROGRAMMING METHODS

Dante Kalise
Imperial College London

Transatlantic flight planning can be cast in the framework of trajectory optimization, dating back to Zermelo's navigation problem (1931). However, in order to study this problem in a realistic setting, different technical challenges must be accounted for. These include, but not limited to: aircraft dynamics, fuel consumption models, and real-time weather information. In this talk, we present recent results in transatlantic flight planning using computational optimization methods. In particular, it is shown that dynamic programming methods can lead to robust, fuel-optimal routes, which can incorporate real-time weather variations in the planning.

GRADIENT BASED ROBUST DESIGN OPTIMIZATION OF ELECTRICAL MACHINES WITH IGA

Theodor Komann
TU Darmstadt

We investigate a nonlinear constrained optimization problem with uncertain parameters for an electrical machine. By utilizing a robust worst-case formulation we obtain an optimization problem of bi-level structure. This type of problems are difficult to treat numerically and hence suitable approximations are required.Our goal is twofold: Firstly, we aim to reduce the volume of the permanent magnet, a significant factor due to its composition of costly rare earth elements. Secondly, we want to minimize the variance of the torque by optimizing the shape of the air gap. Simultaneously, it is crucial to maintain a predetermined performance standard which is given by the torque.The quantities for computing the torque are calculated from magnetic vector potential provided by the magnetostatic approximation of Maxwell's equation, an elliptic PDE. We employ IGA using the open-source GeoPDEs software for the discretization of the PDE. To conclude,we present numerical results.

SAMPLING WITH MOLLIFIED INTERACTION ENERGY DESCENT

Anna Korba
CREST-ENSAE

Sampling from a target measure whose density is only known up to a normalization constant is a fundamental problem in computational statistics and machine learning. We present a new optimization-based method for sampling called mollified interaction energy descent (MIED), that minimizes an energy on probability measures called mollified interaction energie (MIE). The latter converges to the chi-square divergence with respect to the target measure and the gradient flow of the MIE agrees with that of the chi-square divergence, as the mollifiers approach Dirac deltas. Optimizing this energy with proper discretization yields a practical first-order particle-based algorithm for sampling in both unconstrained and constrained domains. We show the performance of our algorithm on both unconstrained and constrained sampling in comparison to state-of-the-art alternatives.

Machine learning model-based outcome optimization in radiotherapy treatment planning

Tim Ortkamp
Karlsruhe Institute of Technology (KIT)

Radiotherapy treatment planning utilizes nonlinear constrained optimization whose components may integrate machine learning models on normal tissue complication probability (NTCP) or tumor control probability (TCP), e.g., as part of the objective function. To facilitate such an integration, we established a Python module for large-scale beam intensity optimization using a nonlinear interior-point quasi-Newton algorithm, and coupled the algorithm with customized feature and model implementations using automatic differentiation. We prototyped three machine learning NTCP models (logistic regression, neural network, support vector machine), and embedded the respective prediction functions in the weighted-sum method for multi-objective optimization. Integration of the machine learning models translated into lower NTCP estimates for all optimized treatment plans, showing the feasibility of the integration step and promising initial results for future application.

COMPUTING MULTIPLE SOLUTIONS OF TOPOLOGY OPTIMIZATION PROBLEMS

Ioannis Papadopoulos
Imperial College London

Topology optimization is heavily used in the design phase of many large-scale engineering projects. Its goal is to find the optimal material distribution of a fluid or solid in a domain, subject to PDE and volume constraints. The models often result in a PDE, volume and inequality constrained, nonconvex, infinite-dimensional optimization problem that may support many local minima. In practice, heuristics are used to obtain the global minimum, but these can fail even in the simplest of cases. In this talk, we will introduce the deflated barrier method, a second-order algorithm that solves such problems, has local superlinear convergence, and can systematically discover many of these local minima. We will present examples which include finding 42 solutions of the topology optimization of a fluid satisfying the Navier-Stokes equations and more recent work involving the three-dimensional topology optimization of a fluid in Stokes flow.

PRECONDITIONED ITERATIVE METHODS FOR TIME-DEPENDENT FLUID FLOW CONTROL PROBLEMS

John Pearson
University of Edinburgh

Optimization problems subject to PDE constraints form a mathematical tool that can be applied to a wide range of scientific processes. It is necessary to obtain accurate solutions to such problems within a reasonable CPU time, particularly for time-dependent problems, for which the "all-at-once" solution can lead to extremely large linear(ized) systems. In this talk we consider iterative methods, in particular Krylov subspace methods, to solve instationary fluid flow control problems where the Stokes or Navier-Stokes equations form the PDE constraints. We employ a Picard (or Oseen-type) outer iteration for Navier-Stokes problems, and derive fast and robust preconditioners for the flexible GMRES method when solving the large-scale systems of equations that result. We employ an inner solver to the leading block of these systems, and to approximate the Schur complement we devise a block-version of a commutator argument used for matrices involving products of divergence and gradient terms.

SOLVING METHODS FOR MIXED-INTEGER NONLINEAR OPTIMIZATION OF HEATING NETWORKS WITH TIME-COUPLED COMPONENTS

Lea Rehlich
TU Darmstadt

To reduce carbon emissions the transformation of the heating sector is of great importance. This leads to a resulting increase of complexity of the heating networks which makes the development of adequate operation strategies more challenging. We consider a global optimization approach that aims at finding cost-optimal operation strategies. The optimization problem is based on nonlinear physical equations for describing the network state and binary variables to determine flow directions in the network pipes. We focus on the stationary case that can be coupled over time via a heating storage. The resulting optimization problem is solved with the solver SCIP. To lower the arising computational costs we consider methods to reduce the number of binary variables and nonlinear equations based on the underlying network structure. In order to solve problems with time coupling we develop a two stage approach. We evaluate our developed methods with numerical results based on real networks.

Using Transport Equations in Image and Shape Registration

Stephan Schmidt
Universität Trier USt-ID: DE 149 881 695

We consider the use of transport equations as a model for solving registration problems. In particular, we differentiate transport equations to determine the best optical flow between different brain scans. In a follow-up procedure, this registration data is used to make a default generic brain mesh patient specific. Special attention is given to treating the non-differentiabilities of the DG0-discretization of the transport equation in regards to the upwind flux.

The turnpike property for mean-field optimal control problems

Chiara Segala
RWTH Aachen University

We study the turnpike phenomenon for optimal control problems with mean field dynamics that are obtained as the limit $N \longrightarrow \infty$ of systems governed by a large number N of ordinary differential equations. We show that the optimal control problems with with large time horizons give rise to a turnpike structure of the optimal state and the optimal control. For the proof, we use the fact that the turnpike structure for the problems on the level of ordinary differential equations is preserved under the corresponding mean-field limit.

MULTI-OBJECTIVE OPTIMIZATION IN TRAFFIC EMISSION MODELING

Alena Ulke
University of Mannheim

We present a multi-objective optimization problem that aims to optimize speed limits on a road network to maximize traffic flow, while simultaneously minimizing the air pollution caused by vehicular traffic. This approach is motivated by the omnipresence of vehicular traffic combined with its negative influence on the environment.

First, we model the underlying dynamic by introducing the framework of a so-called speed-limit-dependent traffic emission model. This framework couples a system of hyperbolic conservation laws to describe the traffic on a road network, an emission model, and an advection-diffusion equation to describe the production and spread of air pollutants. Then we solve the resulting multi-objective optimization problem by a first-discretize-then-optimize approach combined with a pattern search and provide a proof-of-concept example to illustrate the effectiveness of the proposed idea.

PDE-Constraint Optimization for Extended Discontinuous Galerkin and High-order shock fitting

Jakob Vandergrift
TU Darmstadt

We would like to present a high-order shock fitting approach based on a cut-cell method for compressible flows. In our extended discontinuous Galerkin (XDG) method, shocks are sharply represented by the zero-iso contour of a Level Set function that must be aligned with the unknown position of the shock/discontinuity to obtain high-order convergence. To achieve this, we formulate a PDE-constraint optimization problem by introducing an error-based objective function that comes from enriching the test space for the discretized conservation law. The residual from the XDG discretization is, in turn, used as a constraint. We solve the problem using a Quasi-Newton SQP method with partial identity regularization and line search globalization. Additional robustness measures are included, motivated by observations. Lastly, we show results for different 2D steady conservation laws with discontinuities in the solution, where the respective surfaces are successfully tracked.

Domain Decomposition Methods for the Optimal Control of Transport Networks

Lukas Wolff
Universität Mannheim

Optimal control problems, specifically on networks, arise in a variety of applications. They can however pose a significant computational challenge, particularly on large domains or for complex network layouts. We present an approach to address these challenges by employing domain decomposition methods. Each step of the domain decomposition method is combined with a gradient descent procedure into a single iterative scheme, where the forward and adjoint problems are solved in parallel on a number of subdomains. By doing so, we significantly reduce the numerical complexity associated with these problems. For abstract linear problems, we establish a convergence result that can be applied to a wide range of settings. We demonstrate this in the context of hyperbolic transport networks and give numerical illustrations for applications in power grids, gas networks and conveyor belt systems.

8 Optimization on Manifolds (MAN)

TIME-VARYING SEMIDEFINITE PROGRAMMING: PATH FOLLOWING A BURER-MONTEIRO FACTORIZATION

Antonio Bellon
Universität Augsburg

We present an online algorithm for time-varying semidefinite programs (TV-SDPs), based on the tracking of the solution trajectory of a matrix factorization, also known as the Burer-Monteiro factorization, in a path-following procedure. There, a predictor-corrector algorithm solves a sequence of linearized systems. This requires the introduction of a horizontal space constraint to ensure the local injectivity of the factorization. The method produces a sequence of approximate solutions for the original TV-SDP problem, for which we show that they stay close to the optimal solution path if properly initialized. Numerical experiments for a time-varying Max-Cut SDP relaxation demonstrate the computational advantages of the proposed method for tracking TV-SDPs in terms of runtime compared to off-the-shelf interior point methods.

OPTIMIZATION ON THE MANIFOLD OF SYMMETRIC QUASI-DEFINITE MATRICES

Thomas Bendokat
Max Planck Institute for Dynamics of Complex Technical Systems

Symmetric quasi-definite (SQD) matrices feature a block structure with a symmetric positive-definite block and a symmetric negative-definite block on the diagonal. They arise in applications such as interior point methods and determine indefinite inner products. The set of all SQD matrices forms a manifold, which can be studied by means of Lie theory as a quotient space. We make use of this structure to devise some optimization methods and study some numerical examples.

THE DIFFERENCE OF CONVEX ALGORITHM ON RIEMANNIAN MANIFOLDS

Ronny Bergmann
NTNU - The Norwegian University of Science and Technology

In this talk we consider a Riemannian version of the difference of convex algorithm (DCA) to solve a minimization problem for cost functions that consist of a difference of convex functions. We investigate the algorithm, that is its well-definedness, convergence properties and its relation to Riemannian duality. We further present some numerical examples within the Julia packages Manopt.jl and Manifolds.jl.

OPTIMIZATION OF STRICT SADDLE FUNCTIONS AND THE PHASE RETRIEVAL PROBLEM

Florentin Goyens

Many problems in data science can be expressed as the minimization of a strict saddle function. Strict saddle functions are in general nonconvex, but have the property that the Hessian has a negative eigenvalue at every saddle point. Using this information on the landscape, we design an efficient and specific algorithm for the minimisation of strict saddle functions. We derive guarantees on the number of iterations that have to be performed in the worst-case before an approximate minimizer of the function is reached. We show how our algorithm improves on the classical complexity results of nonconvex optimization and illustrate our results on the phase recovery problem.

OPTIMIZATION ON THE SYMPLECTIC STIEFEL MANIFOLD

Rasmus Jensen
University of Southern Denmark (SDU)

In this talk, we will consider a realization of the Riemannian nonlinear conjugate gradient method (CG) on the symplectic Stiefel manifold SpSt(2n,2k). We will discuss the choice of retraction, vector transport, and parameter settings for this iterative method. As an explicit application example, we treat the computation of a Proper Symplectic Decomposition (PSD) of a given data symplectic matrix. This is an important problem in optimization over the symplectic Stiefel manifold SpSt(2n,2k), since it plays a key role, for example, in model reduction of Hamiltonian systems. We compare the convergence behavior of CG with a Riemannian steepest descend method and discuss possible points of improvement.

LIPSCHITZ CONSTANTS BETWEEN RIEMANNIAN METRICS ON THE STIEFEL MANIFOLD

Simon Mataigne
UCLouvain

We give the best Lipschitz constants between the distances induced by any two members of a previously proposed one-parameter family of Riemannian metrics on the Stiefel manifold of orthonormal p-frames in $\mathbb{R}^n$. The one-parameter family contains the well-known canonical and Euclidean metrics. We also provide a lower and an upper bound on these distances in terms of the easy-to-compute Frobenius distance. These bounds yield the equivalence of the distances induced by the family of Riemannian metrics and the Frobenius distance. Moreover, we obtain the diameter of the Stiefel manifold under the Euclidean metric. Finally, we find families of pairs of matrices reaching the upper and lower bounds. These bounds aim at improving the theoretical guarantees and performances of minimizing-geodesic computation algorithms by reducing the size of the initial velocity search space.

MANIFOLD OPTIMIZATION IN DATA FUSION

Max Pfeffer
Uni Göttingen

In Data Fusion, one analyzes different measurements of the same phenomenon, which are stored in several matrices and/or tensors. The goal is to simultaneously factorize these datasets in order to allow for dimensionality reduction, clustering, feature extraction, prediction and the like. The factorizations are often subject to manifold constraints which model the type of data at hand or help with improving the interpretability of the results. In this talk, we discuss several applications for manifold optimization in Data Fusion. We also motivate the necessity of allowing for additional, possibly nonsmooth constraints.

RIEMANNIAN SHAPE OPTIMIZATION OF THIN SHELLS USING ISOGEOMETRIC ANALYSIS

Rozan Rosandi
RPTU Kaiserslautern-Landau, Standort Kaiserslautern

Structural optimization is concerned with finding an optimal design for a structure under mechanical load. In this contribution, we consider thin elastic shell structures [1] based on a linearized shell model of Koiter's type, whose shape can be described by a surface embedded in three-dimensional space. We regard the set of unparametrized embeddings of the surface as an infinite-dimensional Riemannian shape manifold [2] and perform optimization in this setting using the Riemannian shape gradient [3]. Non-uniform rational B-splines (NURBS) are employed to discretize the middle surface and numerically solve the underlying equations that govern the mechanical behavior of the shell via isogeometric analysis [4]. By representing NURBS patches as B-spline patches in real projective space, NURBS weights can also be incorporated into the optimization routine. We discuss the practical implementation of the method and demonstrate our approach on the compliance minimization of a half-cylindrical shell under static load and fixed area constraint. For numerical experiments, we use the GeoPDEs package [5] in MATLAB, extended by the computation of shape sensitivities and Riemannian shape optimization methods.

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Gauss-Southwell type descent methods for low-rank matrix optimization

André Uschmajew
University of Augsburg

We consider gradient-related methods for low-rank matrix optimization with a smooth strongly convex cost function. The methods operate on single factors and share aspects of both alternating and Riemannian optimization. We compare two possible choices for the search directions based on Gauss-Southwell type selection rules: one using the gradient of a factorized non-convex formulation, the other using the Riemannian gradient. Both methods provide convergence guarantees for the gradient that are analogous to the unconstrained case.

APPROXIMATING FUNCTIONS INTO MANIFOLDS FROM SAMPLES

Hang Wang
KU Leuven

A manifold-valued function takes values from a Euclidean domain into a manifold. Approximating a manifold-valued function from samples consists of modeling the relationship between an output on a Riemannian manifold and the Euclidean input vector. In this talk, I will present techniques for approximating a known or unknown manifold-valued function, relying on optimization on manifolds. One approach is based on one tangent space, and the other uses multivariate general linear models. The effectiveness of our models is illustrated with numerical experiments.

9 Optimization under Uncertainty (OU)

DECENTRALIZED LOCAL STOCHASTIC EXTRA-GRADIENT FOR VARIATIONAL INEQUALITIES

Pavel Dvurechenskii
Weierstrass Institute for Applied Analysis and Stochastics (WIAS)

We consider distributed stochastic variational inequalities (VIs) on unbounded domains with the problem data that is heterogeneous (non-IID) and distributed across many devices. We make a very general assumption on the computational network that, in particular, covers the settings of fully decentralized calculations with time-varying networks and centralized topologies commonly used in Federated Learning. Moreover, multiple local updates on the workers can be made for reducing the communication frequency between the workers. We extend the stochastic extragradient method to this very general setting and theoretically analyze its convergence rate in the strongly-monotone, monotone, and non-monotone (when a Minty solution exists) settings. As a special case, our method and analysis apply to distributed stochastic saddle-point problems (SPP), e.g., to the training of Deep Generative Adversarial Networks (GANs). Joint work with A. Beznosikov, A. Koloskova, V. Samokhin, S. Stich, A. Gasnikov

ROBUST RADIOTHERAPY PLANNING WITH SPATIALLY BASED UNCERTAINTY SETS

Noam Goldberg

Radiotherapy treatment planning is a challenging large-scale optimization problem plagued by uncertainty. We propose a novel, spatially based uncertainty set for robust optimization modeling of radiotherapy planning, producing solutions that are immune to changes in biological conditions. Our proposed uncertainty set captures biological radiosensitivity patterns that are observed using recent advances in imaging, while its parameters can be personalized for individual patients. We exploit the structure of this set to devise a compact reformulation of the robust model. We develop a row-generation scheme to solve real, large-scale radiotherapy planning instances of the robust model. The formulation and algorithm are then extended to a relaxation-based scheme to enable enforcement of challenging yet clinically important dose-volume cardinality constraints. The computational performance of our algorithms, as well as the quality and robustness of the computed treatment plans, are demonsted

OPTIMAL CONTROL WITH LARGE PROBABILITY OF FEASIBILITY

Martin Gugat
FAU Erlangen Nürnberg

Often it is desirable to control a system in such a way that the probability that the state remains within the feasible region is maximized (e.g. in the operation of gas networks the feasible region is defined by prescribed lower bounds for the gas pressure). To achieve this goal in an efficient way, it makes sense to state this desire in a probabilistic sense: It is required that the the probability that the pressure always remains above the given bound is as large as possible. To obtain controls that achieve this goal, we consider optimal control problems where the objective function contains the probability of feasibility. To guarantee the existence of optimal controls, the objective functional also contains an additional regularization term that can be regarded as the control cost. The system dynamics is governed by the isothermal Euler equations. The control acts through the boundary conditions. The consumer demand is unknown and can be considered as a random variable.

ENSEMBLE FEEDBACK STABILIZATION OF LINEAR SYSTEMS

Philipp A. Guth
Austrian Academy of Sciences

A Riccati-based feedback mechanism is analyzed for stabilization of linear control systems with parameter-dependent system matrices. The feedback is constructed by means of a finite ensemble of parameters from a training set. Conditions are provided under which this feedback law stabilizes all systems of the training set and also systems in its vicinity. Moreover, its suboptimality with respect to the optimal feedback for each single parameter from the training set can be quantified. The theoretical findings are illustrated with a set of application-motivated examples and confirming numerical experiments.

Robust Variational Control Problem with Isoperimetric Constraints

Anurag Jayswal
Indian Insitiute of Technology (ISM) Dhanbad

The aim of this paper is to focus study on an uncertain variational control problem with isoperimetric constraints and its robust counterpart. For this, we first transform the isoperimetric constraints into a system of differential equations using an auxiliary variable. Later on, we derive the robust necessary optimality conditions for the aforementioned problem and its sufficiency utilizing the concept of convex functions. Also, we define the robust saddle point for the corresponding uncertain Lagrange function and establish its equivalence with the robust optimal solution of the problem under study. In addition, some numerical applications are formulated to validate the stated results.

GENERAL POLYHEDRAL APPROXIMATION OF TWO-STAGE ROBUST LINEAR PROGRAMMING

Tim Niemann
Technische Universität Braunschweig

In this talk, we consider two-stage robust linear programs with a budget of uncertainty for the righthand side. We develop a General Polyhedral Approximation (GPA) method, in which the uncertainty set is substituted by a finite set of polytopes derived from the vertices of an arbitrary polytope that dominates the uncertainty set. The method combines the solutions for the individual polytopes and thereby achieves an approximation factor that significantly improves on the literature while sustaining practical running times. Moreover, the method allows for even stronger results on specific problems as we exemplify for the Transportation Location Problem. Previous piecewise affine methods reach a threshold at the square root of the dimension of the righthand side after which they are not better than a quasi nominal solution. The GPA method enhances the approximation factor prior to this threshold and offers a trade-off between the number of vertices and the approximation factor beyond it.

An a posteriori Probabilistic Robustness Check for Deterministic Optimal Controls

Michael Schuster
FAU Erlangen-Nürnberg

We consider uncertain gas transport in pipeline networks and we propose a novel strategy to measure the robustness of determinstically computed compressor controls. We first consider a deterministic gas network and optimize the control cost w.r.t certain constraints on the pressure. Then we consider uncertain gas demands, but maintain the determinstically computed control. We define the probabilistic robustness of the deterministic control as the probability that the pressure satisfies given bounds. Moreover, we analyze the influence of buffer zones for the pressures w.r.t. the probabilistic robustness of the control. For the computation of the probability, we apply a kernel density estimator based on samples of the uncertain pressure combined with an adaptive stochastic collocation method to approximate the pressure in the stochastic space. We discuss generalizations of the probabilistic robustness check and we present numerical results for real-world based gas network instances.

STOCHASTIC RESOURCE ALLOCATION PROBLEMS WITH DETERMINING RECOURSE: FORMULATIONS AND COMPLEXITY

Bismark Singh
University of Southampton

Motivated by the general problem of allocating resources to users, we present a two-stage stochastic optimization model and study its theoretical properties. The first-stage decision variables determine two types of policies that we formally distinguish: either release resources immediately or release resources sequentially. We show that following one policy when the other is provably optimal can be arbitrarily bad. The second-stage decision variables provide a new general class of recourse functions for stochastic programs that we coin as determining recourse; here, all feasible second-stage decisions are also optimal for the recourse function. Determining recourse is a subset of relatively complete recourse and a superset of simple recourse. Special cases of our problem reduce to well-known knapsack problems and we prove that the underlying problem is weakly NP-complete.

A REGULARIZED VARIANCE-REDUCED MODIFIED EXTRAGRADIENT METHOD FOR STOCHASTIC HIERARCHICAL GAMES

Mathias Staudigl
Mannheim University

The theory of learning in games has so far focused mainly on games with simultaneous moves. Recently, researchers in machine learning have started investigating learning dynamics in games involving hierarchical decision-making. We consider an N-player hierarchical game in which the i-th player's objective comprises of an expectation-valued term, parametrized by rival decisions, and a hierarchical term. Such a framework allows for capturing a broad range of stochastic hierarchical optimization problems, Stackelberg equilibrium problems, and leader-follower games. We develop an iteratively regularized and smoothed variance-reduced modified extragradient framework for learning hierarchical equilibria in a stochastic setting. We equip our analysis with rate statements, complexity guarantees, and almost-sure convergence claims. We then extend these statements to settings where the lower-level problem is solved inexactly and provide the corresponding rate and complexity statements.

CONTINUOUS-STOCHASTIC METHODS FOR OPTIMIZATION UNDER UNCERTAINTY

Andrian Uihlein
Friedrich-Alexander-Universität Erlangen-Nürnberg

In large scale optimization with expected valued objectives, traditional deterministic approaches are attached to high computational costs and introduce unwanted discretization errors into the problem. In contrast, standard stochastic techniques provide weaker convergence results and are typically limited to expected values entering the objective function linearly. As a hybrid of these concepts, we propose continuous-stochastic schemes. To keep the numerical effort of iterations low, CS methods work with random samples instead of full expectations. Contrary to most other sample-based approaches, samples are not discarded after a step is performed. Instead, old samples are adaptively recombined to obtain an approximation to the full objective gradient. We show that the approximation error converges to zero during the optimization process. As a result, CS methods inherit strong convergence results known from deterministic optimization and allow for a wider class of possible objectives.

RANDOMIZED ZEROTH-ORDER METHODS FOR NONDIFFERENTIABLE, NONCONVEX, AND HIERARCHICAL FEDERATED OPTIMIZATION

Farzad Yousefian
Rutgers University

Federated learning (FL) has recently emerged as an enabling framework for communication-efficient decentralized training. We study the following broadly applicable problem classes in FL: (i) Nondifferentiable nonconvex optimization, (ii) Federated bilevel optimization (iii) Federated minimax problems. Most existing results are reliant on strong assumptions, including the differentiability and L-smoothness of the objective function. Unfortunately, such assumptions may fail to hold in practical settings. We bridge this gap as follows. In (i), by leveraging convolution-based smoothing and Clarke's subdifferential calculus, we devise a randomized zeroth-order FL method and derive complexity guarantees for computing an approximate Clarke stationary point. In (ii) and (iii), we devise a unifying randomized implicit zeroth-order FL method with explicit communication and iteration complexities. We validate the theory using numerical experiments on nonsmooth and hierarchical ML problems.

A STOCHASTIC GRADIENT ALGORITHM WITH MOMENTUM TERMS FOR OPTIMAL CONTROL PROBLEMS CONTAINING RANDOM INPUT DATA

Hamdullah Yücel
Middle East Technical University (METU)

In this talk, we focus on a numerical investigation of a strongly convex and smooth optimization problem subject to a convection diffusion equation with random input data. Considering that Monte Carlo approximation is a finite summing, we replace the true gradient by a stochastic ones with suitable momentum terms, that is, Polyak's or Nesterov's momentum terms, to minimize the objective functional containing random terms. Step size and momentum terms are chosen in an adaptive framework. The efficiency of the proposed methodologies will be illustrated by the benchmark problems.

10 Shape and Topology Optimization (STO)

DENSITY-BASED TOPOLOGY OPTIMIZATION OF WAVEGUIDE ACOUSTIC BLACK HOLES

Martin Berggren
Umeå Universitet

The acoustic black hole effect is a slowing down of the effective propagation velocity of waves and an associated focusing of the wave power. It is known how to design acoustic black hole devices for beams and plates. However, no design has so far been found that achieves a broadband focusing effect for sound waves in air. Here we apply topology optimization that distributes solid material in a region to focus the sound pressure at the end of a waveguide. To model viscothermal boundary-layer losses, we use a recently devised computationally inexpensive custom boundary condition. By tracking the contrasts between cells in the design variables, the boundary condition will be applied only at high contrast levels. Using the classic gradient-based MMA algorithm together with a quadratic penalty for the optimization variables and a nonlinear filter strategy to suppress scattered solid material, we are able to design what we believe is the first true focusing waveguide acoustic black hole.

OPTIMAL CONTROL OF ANISOTROPIC ALLEN-CAHN EQUATIONS

Luise Blank
University of Regensburg

Anisotropic Allen-Cahn equations model for example the interface evolution in crystal growth, which shall be controlled. To treat the anisotropy nondifferentiable terms we propose a specific regularization. The issue of differentiability leads us to choose the implicit time discretization where also energy stability is obtained.

Lipschitz-continuity of the control-to-state operator is shown for the time discretized and the time continuous problem. The existence of a global minimizer of the original and of the regularized problem is provided. Also the convergence with respect to regularization and to discretization is considered. For the regularized problem the Fréchet differentiability is obtained.

The trust-region Newton Steihaug-cg method is applied to the time discretized problem which is then discretized in space. This leads to iteration numbers bounded independently of the discretization level. Numerical examples are presented.

MOMENTUM BASED NON-CONVEX SHAPE OPTIMIZATION UTILIZING DISSIPATIVE HAMILTONIAN FLOWS

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University of Wuppertal, Faculty of Mathematics and Natural Sciences

Shape optimization with constraints given by partial differential equations (PDE) is a highly developed field of optimization theory. The adjoint formalism allows to compute shape gradients at the computational cost of a further PDE solve. Thus, gradient descent methods can be applied to shape optimization problems. However, gradient descent methods that can be understood as approximation to gradient flows get stuck in local minima, if the optimization problem is non-convex. In machine learning, the optimization in non-convex energy landscapes has been successfully tackled by momentum methods, which can be understood as passing from gradient flow to dissipative Hamiltonian flows. We adopt this strategy for non-convex shape optimization. In particular, we provide a mechanical shape optimization problem that is motivated by optimal reliability considering also material cost and installation space restrictions. We then show how this problem can be solved by dissipative Hamiltonian flows.

DEHOMOGENIZATION IN STRESS MINIMIZATION PROBLEMS

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CIMNE - International Center for Numerical Methods in Engineering

In the last years, additive manufacturing has allowed to build lattice structures with impressive small length scale. This breakthrough has forced the topology optimization community to propose new techniques that are able to consider an immense number of design variables. Some attempts include multi-scale topology optimization techniques, however its prohibitive computational cost has made this technique less attractive from a computational point of view. As an alternative, the dehomogenization method has recently shown very promising results in terms of performance and computational cost for compliance examples. Due to the importance of stresses in structural design, in this work, we extend the compliance results to the stress minimization problems. We also consider singularities in the orientation field by adding singular functions in the dehomogenization process. We finally present several numerical examples to validate the proposed methodology

Lipschitz methods in PDE constrained shape optimization

Michael Hinze
Universität Koblenz

We present a general shape optimisation framework based on the method of mappings in the Lipschitz topology. We propose steepest descent and Newton-like minimisation algorithms for the numerical solution of the respective shape optimisation problems. Our work is built upon previous work of the authors in (Deckelnick, Herbert, and Hinze, ESAIM: COCV 28 (2022)), where a Lipschitz framework for star-shaped domains is proposed. To illustrate our approach we present a selection of PDE constrained shape optimisation problems and compare our findings to results from so far classical Hilbert space methods and recent p-approximations. We also provide numerical convergence analysis.

Sensitivity analysis and tailored design of minimization diagrams

Antoine Laurain
University of Duisburg-Essen

Minimization diagrams encompass a large class of diagrams of interest in the literature, such as generalized Voronoi diagrams. We develop an abstract perturbation theory in two dimensions and perform a sensitivity analysis for functions depending on sets defined through intersections of smooth sublevel sets, and formulate precise conditions to avoid singular situations. This allows us to define a general framework for solving optimization problems depending on two-dimensional minimization diagrams. The particular case of Voronoi diagrams is discussed to illustrate the general theory. A variety of numerical experiments is presented, which show that the proposed methodology allows the construction of customized Voronoi diagrams using off-the-shelf well-established optimization algorithms.

ROCK MASS PROPERTIES BY THE STOCHASTIC HOMOGENIZATION METHOD

BOCHRA MEJRI
Radon Institute, Austrian Academy of Sciences, RICAM Linz

This study concerns the determination of the elastic properties of intensely fractured rocks around tectonic faults. The objective is to derive a mathematical compliance modeling of naturally fractured rock mass. A two-stage procedure is applied. The first one was consecrated to elaborate an inferential statistical analysis of the geometric parameters delineating fracture networks. The statistical data are extracted from image analysis (Faults R Gems project) in situ Granite Dells in Prescott, Arizona. An algorithm is developed to generate synthetic fracture networks based on the derived statistical laws. The second stage was devoted to derive the elastic properties of the fractured rock masses. Therefore, their overall mechanical behavior is modeled as a homogenized macroscopic equivalent medium. A dual boundary integral equation (BIE) formulation using conventional BIE and hyper-singular BIE is applied to model fractures in two-dimensional linear elastic solids.

MESH QUALITY WITH A NONLINEAR EXTENSION EQUATION

Sofiya Onyshkevych
Universität Hamburg

During shape optimization we have to update the geometry iteratively. In the presence of large deformations this process often results in a loss of mesh quality and even degenerate meshes.

We prefer to avoid re-meshing and model the mesh deformation using the method of mappings. To reduce computational costs and promote the preservation of mesh quality, we use an extension equation that maps a boundary control variable to a deformation field defined over the entire domain. By using the nonlinear extension operator proposed in previous work, we increase the set of reachable shapes, allowing us to model large deformations.

In this talk, we discuss how the choice of parameters of the extension equation affects the mesh quality. Furthermore, we investigate the influence of the extension factor by studying the condition number of the Hessian of the optimality system derived from the Lagrangian.

OPTIMIZATION OF PIECEWISE-SMOOTH SHAPES: AN APPROACH FROM RIEMANNIAN GEOMETRY

Lidiya Pryymak

The design of structures plays a crucial role in many engineering applications, such as cardiovascular stents and aerodynamic wings. Hereby, optimal designs can be derived using a shape optimization formulation of the problem. Shape optimization is highly dependent on the choice of the underlying shape space, i.e., a space in which each element is given by a shape. Often, it is convenient to choose the shape space as a Riemannian manifold. In this case, the design of an optimization algorithm is based on techniques from differential geometry. For many considered shape spaces, challenges arise when it is necessary to optimize a non-smooth shape, which is encountered as an optimal shape for fluid-mechanical problems. In this talk, we present the novel space of piecewise-smooth shapes, which has the structure of a Riemannian product manifold. Moreover, we investigate the optimization of multiple shapes. Finally, we present numerical results for a fluid-mechanical problem.

DETECTION OF MULTIPLE IMPEDANCE OBSTACLES BY TOPOLOGICAL IMAGING METHODS

Maria-Luisa Rapun
Universidad Politécnica de Madrid

In this work we investigate iterative imaging techniques for multiple 2D and 3D acoustic obstacles fully-coated by a complex surface impedance. Closed-form expressions of the obstacle indicator function will be derived. New imaging algorithms either fully or partially based on topological derivative computations will be presented. The performance of the methods will be illustrated in a wide variety of numerical examples.

INTERFACE IDENTIFICATION CONSTRAINED BY NONLOCAL EQUATIONS

Matthias Schuster
Trier University

Models of physical phenomena that use nonlocal operators are better suited for some applications than their classical counterparts that employ partial differential operators. However, the numerical solution of these nonlocal problems can be quite expensive. Therefore, Local-to-Nonlocal couplings have emerged that combine partial differential operators with nonlocal operators. In this talk, we make use of an energy-based Local-to-Nonlocal coupling that serves as a constraint for an interface identification problem. Moreover, we will take a look at the second shape derivative for the interface identification problem constrained by nonlocal equations.

ERROR-CONTROLLED SHAPE OPTIMIZATION BY CONSTRAINED FIRST-ORDER SYSTEM LEAST SQUARES

Gerhard Starke
Universität Duisburg-Essen

This contribution is concerned with computational aspects of PDE-constrained shape optimization problems. A computable estimator for the shape stationarity measured by the dual norm of the distributed form of the shape derivative is derived and shown to accurately reflect the distance from shape optimality. The evaluation of this estimator involves the solution of a constrained best approximation problem in matrix-valued H(div) spaces. This is related to the tensor representation of the distributed shape derivative elaborated in [LauStu:16]. Matrix-valued Raviart-Thomas spaces are used for its computational realization. The shape gradient is recovered from this formulation as a Lagrange multiplier. We illustrate the viability of this approach by computational results for a number of common examples of shape optimization problems.

[LauStu:16] A. Laurain, K. Sturm: Distributed shape derivative via averaged adjoint method and applications. ESAIM Math. Model. Numer. Anal. 50, 2016.

ON A NEW CLASS OF SEPARABLE MODELS IN TOPOLOGY AND MATERIAL OPTIMIZATION

Michael Stingl
Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU)

Separable models are used in specialized optimization solvers for structural optimization like the method of moving asymptotes (MMA) for many years. In this presentation, a new derivation of so called exact separable models using the Sherman-Morrison-Woodbury-Formula is discussed. In this context, exact means that the model matches an approximated function exactly, as long as the design of only one element is varied. In simple cases, there is a 1-to-1 correspondance to the MMA model. In more general situations, non-linear material parametrizations cause the exact separable model to be non-convex. In this case, the sequential global programming (SGP) algorithm is used to solve to provide globally optimal solutions. Throughout the presentation, exact separables are introduced, approximations, e.g., based on the concept of topological derivatives are shown and applications from various areas of topology and material optimization are used to demonstrate their efficiency.

A TOPOLOGICAL DERIVATIVE-BASED ALGORITHM TO SOLVE OPTIMAL CONTROL PROBLEMS WITH $L^0(\Omega)$ CONTROL COST

Daniel Wachsmuth
University of Wuerzburg

In this paper, we consider optimization problems with L^0-cost of the controls. Here, we take the support of the control as independent optimization variable. Topological derivatives of the corresponding value function with respect to variations of the support are derived. These topological derivatives are used in a novel gradient descent algorithm with Armijo line-search. Under suitable assumptions, the algorithm produces a minimizing sequence.